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PHYSICS OF TECHNOLOGY

COORDINATED BY AMERICAN INSTITUTE OF PHYSICS

THE INCANDESCENT LAMP

Thermodynamics, Current Electricity, and Photometry.

THE INCANDESCENT LAMP

A Module on Thermodynamics, Current Electricity, and Photometry

FVCC

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The Incandescent Lamp

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THE INCANDESCENT LAMP

INTRODUCTION

In this module, the incandescent lamp is used as a device to help you learn principles of heat radiation and the measurement of light. You may find it surprising that so many different principles are involved in the operation of such an apparently simple and common device. Actually, much more could have been written on the incandescent lamp, but that would have taken too much of your time to study.

In this module, you will find many unfamiliar technical terms. A glossary of definitions is included in an appendix to help you with those terms.

PREREQUISITES

Even though most of the principles you need to know in order to understand the incandescent lamp are presented within the module, there are a few things you should know to start with. These prerequisites include a working knowledge of Ohm's Law and direct current circuit power equations. You should also be able to read meters and make simple circuit connections.

If you can answer all the following items, you are ready to begin the module. If you have trouble with any of the items, get help from your teacher or another student. Your teacher has the correct answers to the items.

1. The following statements describe certain electrical quantities. Which quantity listed most closely refers to each statement?

   Statements

   A. A resistor is connected into a circuit so that it carries a current. When the resistor is immersed in a cupful of water, the water temperature increases at a constant rate. The rate at which this happens is related to the input of ________.

   B. An electric charge passes through a circuit with resistors in it. To pass through the circuit the charge must be pushed by ________.

   C. A battery maintains an electric current in a circuit until the battery is "dead." When this happens, the battery is no longer a source of ________.

   D. A wire carrying an electric current is observed to heat up. The wire has the property of ________.

   E. In an electroplating device, a certain number of charged atoms (ions) reach one of the plates per second. From this number we can calculate how much electric charge passes through the circuit per second. The amount of charge passing per second is ________.

   Quantities

   a. EMF (electromotive force)
   b. Potential difference (voltage)
   c. Electric power
   d. Electric current
   e. Resistance

2. Match the correct unit(s) with the quantity listed. (In some cases, more than one unit is correct.)

   Quantities

   A. EMF
   B. Potential difference
   C. Electric power
   D. Electric current
   E. Resistance

   Units

a.	coulomb	e.	watt
b.	milliampere	f.	volt
c.	ampere	g.	ohm
d.	millivolt	h.	kilowatt

3. Your instructor will give you two resistors, a battery, a switch, and some connecting wire. Connect the resistors in series with the battery and switch, and leave the switch open.

4. Using the circuit you have wired in Item 2, connect an ammeter in the circuit, connect a voltmeter to measure the voltage across one of the resistors, and leave the switch open until your instructor checks your circuit. When he has checked the circuit, close the switch, measure, and record the current and voltage.

5. Using Ohm's Law ($V = IR$) and the values of current and voltage found in Item 3, calculate the resistance of the resistor across which you measured voltage.

6. Using the values of voltage and current you measured in Item 3, calculate the power input to the resistor.

GOALS FOR SECTION A

The following goals state what you should be able to do after you have completed this section of the module. These goals must be studied carefully as you proceed through the module, and as you prepare for the post-test. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal. Answers to the items are found immediately following these goals.

1. *Goal:* Know what the terms continuous spectrum and line spectrum mean, and under what conditions each may be observed.

   Item: Describe the spectrum seen through a prism or diffraction grating when you view light from

   a. an incandescent lamp;
   b. a sodium-vapor lamp.

2. *Goal:* Know which colors are spectral colors and their relative positions in the spectrum.

   Item: Arrange the following colors into a spectrum, discarding non-spectral colors:

   pink, red, purple, blue, green, violet, brown, yellow, orange.

3. *Goal:* Know the definition of wavelength.

   Item: Given a graph of a wave and an appropriate scale, be able to find the wavelength of the wave.

4. *Goal:* Understand the relation between the temperature of a hot object and its color.

   Item: Identical iron bars are heated until one is glowing yellow and one orange. Which would feel warmer when you place your hand several inches to the side of each iron bar?

5. *Goal:* Be able to use a calibrated spectrometer to measure wavelength.

   Item: Given a hand-held calibrated spectrometer and a mercury-vapor lamp, find the approximate wavelength of the green spectral line.

Answers for Items Accompanying the Preceding Goals

1. a. Continuous spread of colors from red to violet.
   b. Distinct, separate, colored images of the source; one yellow, fainter red, green, and violet ones.

2. Red, orange, yellow, green, blue, violet.

3. Use the provided scale to measure the distance from one crest of the wave to the next: this length is the wavelength.

4. The yellow one.

5. With the circular opening to your eye, point the slit toward the lamp. Read the position of the green line on the numerical scale. It should read about 550 nm.

SECTION A

A Qualitative Approach

DESCRIPTION OF AN INCANDESCENT LAMP

The *incandescent lamp,* or light bulb, is the most widely used and familiar source of artificial light. Figure 1 shows the parts of an incandescent lamp. In operation, electric current is passed through a thin wire called the *filament* which is inside the glass bulb, or *envelope.* The filament heats up until it becomes *incandescent* (glows white-hot). Except for the sun or a star, light bulb filaments are among the hottest things you will ever see. The filament is so hot that it would burn up if it were exposed to air. Therefore, the filament is enclosed in its own special environment within the glass bulb.

You are already familiar with heating by an electric current and with the incandescence of a hot solid. The same processes take place in toaster coils and heating elements of electric stoves, as well as in other household devices. However, it is still worthwhile to arrange your observations about such phenomena in some logical order.

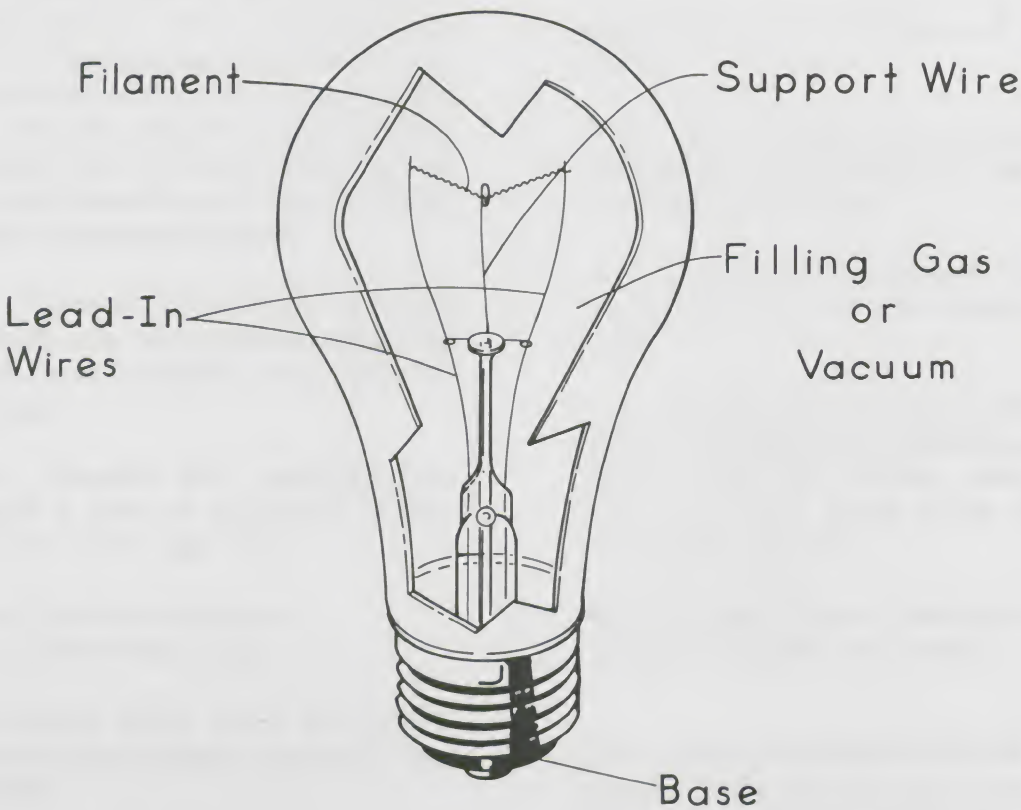


Figure 1.

EXPERIMENT A-1. Emission of Light and Heat by Solids

Part I

For this part of the experiment you will need a bunsen burner, pieces of copper wire and iron wire, a stand for holding the wires in the flame, safety glasses to protect your eyes, and a color comparator (see Figure 2).

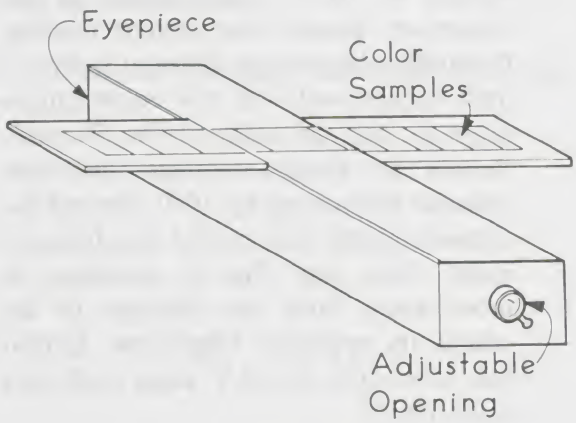


Figure 2.

Fix the iron wire to the stand, turn on the bunsen burner, and move the stand so that one end of the wire is at the upper tip of the flame, as shown in Figure 3.

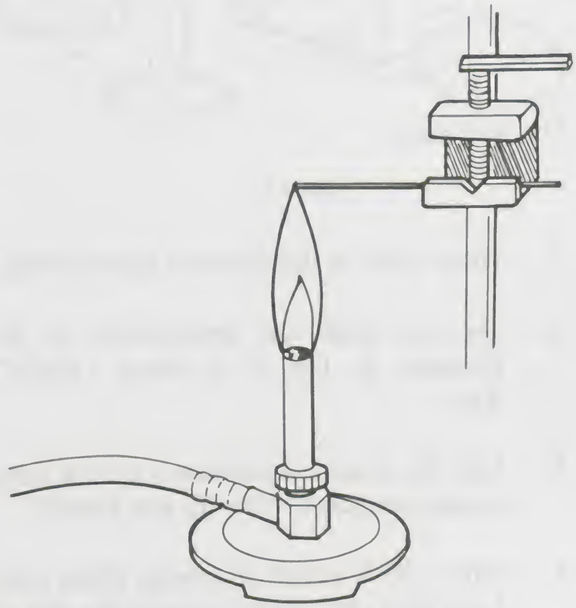


Figure 3.

Answer the following questions on the work sheets provided at the end of the module.

1. Does the iron wire emit (give off) light? If so, what portion of the the wire emits?

2. What color of light is emitted? Is the color different for different portions of the wire? Describe these colors.

3. In order to identify the color at the end of the wire, use the color comparator shown in Figure 2.

   With the color strip on top, look through the eyepiece while aiming the adjustable aperture at the object. You will see an image of the object surrounded by the sample color. It may be necessary to move under a light in order to make the sample bright enough. The color sample can be changed by sliding the color strip through the comparator. It is helpful to match the brightness of the object with the brightness of the sample. The adjustable opening or aperture allows you to do this. Find the color sample which best matches the color at the end of the wire. Record the name and number of the sample.

Now move the wire down so that the end is just above the blue part of the flame, as shown in Figure 4. This is the hottest part of the bunsen burner flame.

4. Describe the difference between the amount of light emitted by the wire now and in the previous position.

5. Describe the difference in the color of the light.

6. Use the color comparator to match the color of the end of the wire with one of the color samples as you did in step 3.

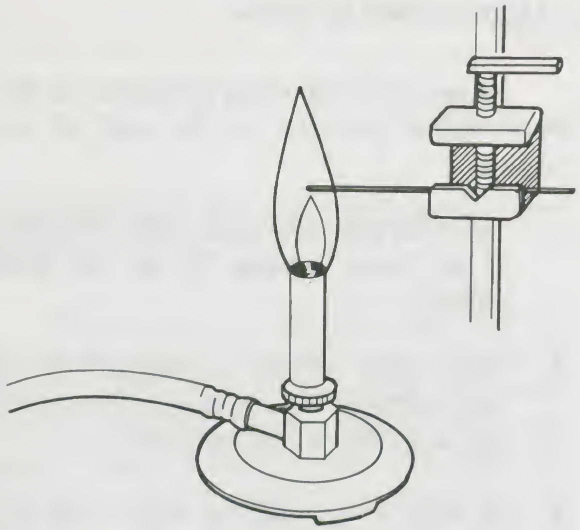


Figure 4.

7. Remove the wire from the flame. Describe the change in the color and in the amount of light emitted as the wire cools down.

Replace the iron wire with the copper wire. **Put on your safety glasses.** Place the copper wire in the flame as shown in Figure 3.

8. Does its appearance differ from that of the iron wire in the same position? If so, how?

9. Match the color again.

Move the copper wire down to the position shown in Figure 4.

10. Does the copper behave any differently than the iron in this position? If so, how?

11. Match the color again.

12. The melting point of copper is 1083°C and that of iron is 1535°C. Can you make a rough estimate of the temperature at the hottest part of the flame?

13. From your observations, try to draw a few general conclusions about the light emitted by a hot solid, and record them.

Part II

Here you are given a clear incandescent lamp with a straight filament. Connect the lamp to a DC power supply which allows you to vary the voltage from 0 to 100 V. Include meters to read voltage and current. The arrangement is shown in Figure 5.

1. Turn on the power supply and set the voltage at 10 V, as indicated on the voltmeter. Record the current reading. Carefully observe the filament against a dark background. Use the color comparator to find the color of the filament. Record the filament voltage and color. Increase the voltage by 10 V. Record the current. Match and record the filament color. You may find it necessary to move away from the filament to decrease its apparent brightness. Repeat this procedure in 10 V steps until you reach 100 V.

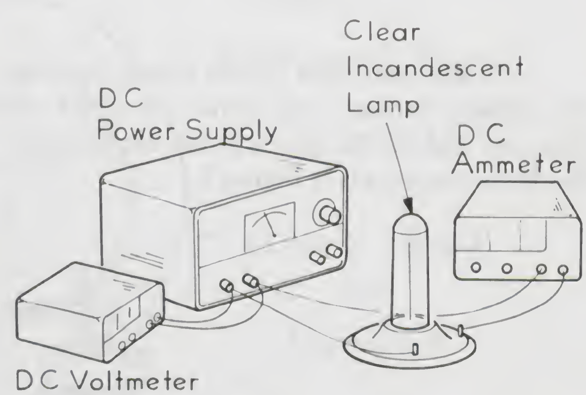


Figure 5.

2. What color is the filament approaching?

3. Do you think the temperature of the filament at 100 V is above 1100°C? Why?

4. Did the electrical resistance of the lamp remain constant? How do you know?

5. With 100 V across the lamp, place your hand near (but do not touch) the side of the glass bulb. What do you feel? What

happens to what you feel when the voltage is rapidly lowered to 10 V?

Part III

Because of the small size of the filaments, it is difficult to feel any heat emitted from them unless they are at high temperatures. For this reason, we shall return to the bunsen burner and heat a larger object: the edge of a ring stand.

1. Turn on the burner and place it so that the edge of the ring stand is in the hottest part of the flame. Keep it there for 5 minutes. Does this heated solid emit light?

2. Remove the burner and place your hand near (but **do not touch**) the heated side of the ring stand. Does the heated solid now emit heat?

3. From your observations, try to draw some general conclusions about the emission of light and heat by a solid.

THERMAL RADIATION

You have seen that a heated solid can emit heat, or heat and light together, depending on its temperature. An object that emits heat or light is said to be *radiating*. Whatever is emitted can be called radiation. The light and heat radiated by a hot solid are called *thermal radiation* because they result from the high temperature of the object.

Actually, the heat and light radiated by a hot solid are very similar to each other. One difference is that one type of radiation is sensed by the skin and the other is sensed by the eye. Either type of radiation is able to travel through empty space (vacuum). You can see that this is true because small (25 W or less) incandescent lamps emit both heat and light. These small lamps have no gas inside the glass bulb, so the filament is in a vacuum.

For objects which are at lower temperatures, the radiation is almost exclusively in the form of heat. As the object gets hotter, more and more of the radiation is in the form of light. This result is used in incandescent lamps, although even at the high temperature of the filament, most of the radiation is still in the form of heat.

Question 1. If you could control the voltage across the incandescent lamp and you wanted its light to be more red, would you turn the voltage up or down?

Question 2. What other observations, besides those on vacuum light bulbs, would lead you to believe that light and heat can be radiated through empty space?

Question 3. Would you estimate the temperature of the surface of the sum to be greater or less than 1100°C? Why?

HISTORY OF THE INCANDESCENT LAMP

Incandescent lamps have been with us for a long time. The first practical light bulb was invented in 1879 by Thomas A. Edison. It was probably his most famous invention. His greatest difficulty was finding a filament material which could withstand the high temperatures involved. Although Edison tried many materials as filaments, the bulbs would burn out too quickly to be useful. His first successful lamp had a carbon filament in a vacuum. One major drawback of carbon filaments is that they *evaporate* rapidly at high temperatures. That is, at temperatures near the melting temperature, some carbon atoms have high enough energies to escape the filament and to move to the glass bulb where they "stick." This causes the inside of the bulb to become black and, of course, reduces the light output of the bulb. It also weakens the filament, which soon breaks.

Since 1910, filaments have been made from tungsten, which has a high melting point and which does not evaporate rapidly. The high melting point of tungsten allows operation of the bulb at a higher temperature, which in turn produces more visible light output for a given electric power input. Another modern improvement was the introduction of a gas into the glass bulb. This gas helps to prevent evaporation of the filament, assuring a longer bulb life. Less evaporation of the filament also means less blackening of the inside of the glass bulb, which keeps the light output from declining as the bulb ages. Usually a mixture of argon and nitrogen is used to fill the bulb, since neither of these gases enters readily into any chemical reaction with the lamp components.

OTHER LIGHT SOURCES

A more recent invention is the fluorescent lamp, which has replaced the incandescent lamp in many commercial, industrial, and institutional applications. You may have noticed that the light from a fluorescent lamp appears whiter than that from an incandescent lamp. Another common observation is that colored objects appear to be slightly different in color when viewed under the two different sources.

Question 4. From its color, would you expect a fluorescent lamp to be hotter or cooler than an incandescent lamp? Why? Do other observations bear out your conclusion?

Still more recently developed sources of light are the mercury-vapor and sodium-vapor lamps, which are often used for outdoor illumination of streets and parking lots. Light from the mercury lamps has a bluish cast and light from the sodium lamps is yellow. Each may produce unexpected color sensations when it is reflected from familiar objects.

Fluorescent and mercury-vapor lamps do not produce light simply by heating a solid to a high temperature, as does the incandescent lamp. When viewed directly, the three sources produce slightly different sensations of white (yellowish-white, bluish-white, etc.). However, you can study the differences in more detail by doing the next experiment.

EXPERIMENT A-2. Analysis of White Light

Part I

You have been provided with an incandescent source of light along with a slit, with which to produce a light beam, and an optical bench. Two identical glass *prisms* and a mounted film called a *diffraction grating* can be placed on the component table.

Place one of the prisms on the table so that it intercepts the beam from the slit as shown in Figure 6.

1. Adjust the angle of the prism and the position of a white screen until you can see a bright band of colors on the screen as shown in Figure 6. What colors do you see?

2. Do you see any white light coming out of the prism?

3. Where did the colors come from?

Place the second prism on the table along with the first, as shown in Figure 7, so that it will bend the light in the opposite direction.

4. Do you see any colored light coming out of the second prism?

5. Do you think that the colors you saw in the previous arrangement were present in the original beam of white light or were they added by the prism? Give reasons for your answer.

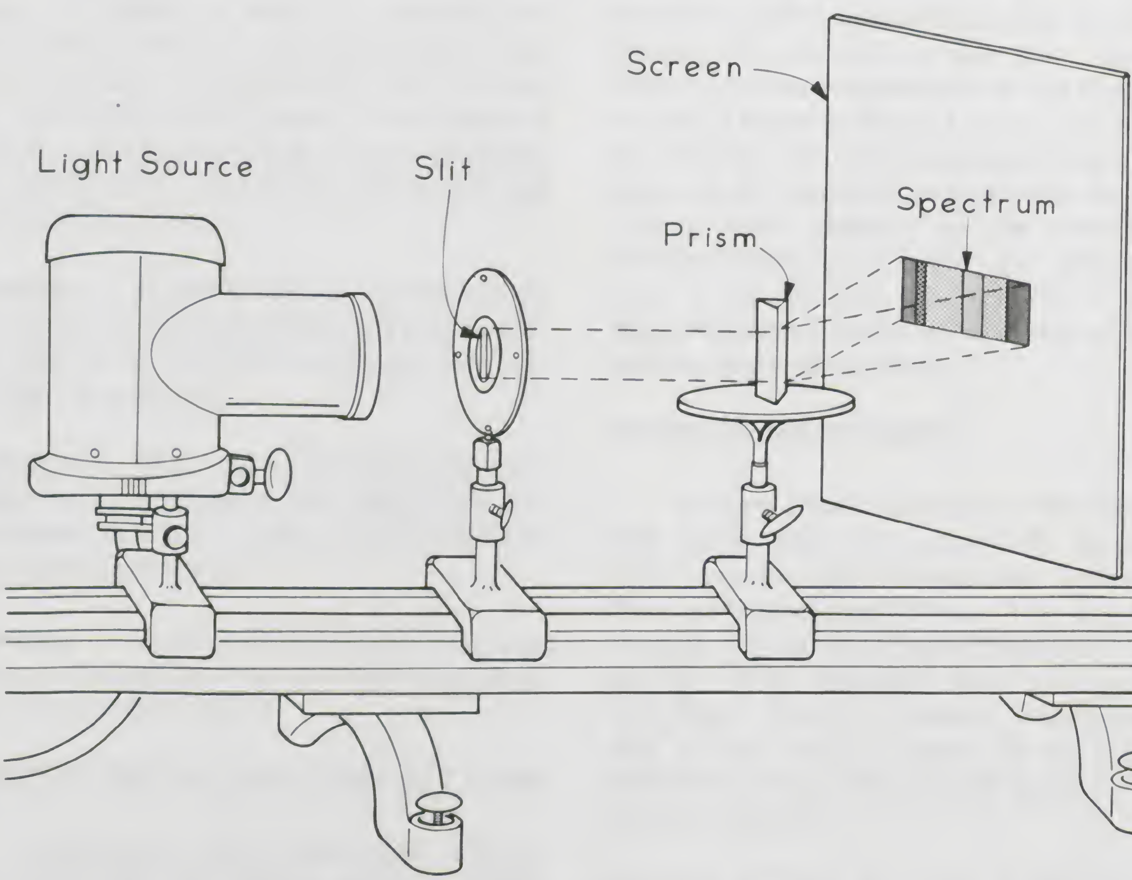


Figure 6.

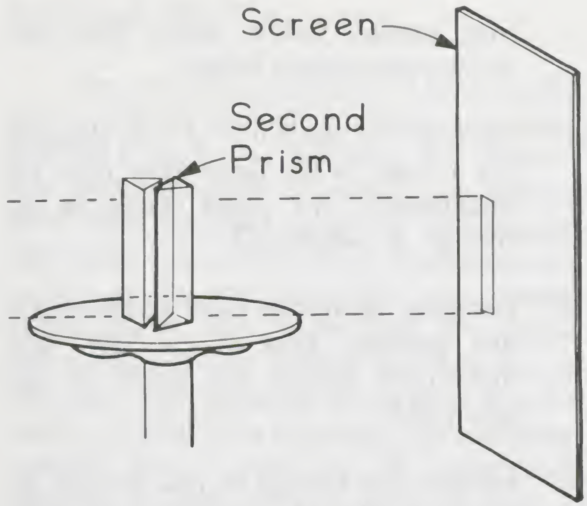


Figure 7.

Now remove the prisms and place the mounted diffraction grating on the table, perpendicular to the white light beam from the slit. Place the white screen behind the diffraction grating and parallel to it as shown in Figure 8.

6. Do you see any white light?

7. What colors do you see?

8. How does the colored band of light compare with that produced by a prism?

The band of colored light you see emerging from a prism or a diffraction grating is called a *spectrum* (plural: *spectra*). A spectrum of a light source is the set of colors in the light which is produced by that source.

Part II

In this part of the experiment we will use a diffraction grating to produce spectra of light sources, because the grating is easier to incorporate into a simple optical instrument. The optical instrument in this case is a small *spectrometer* consisting of a slit and a grating placed at opposite ends of an enclosure. With this instrument you can analyze the white light from three sources: a straight filament incandescent lamp, a straight mercury-vapor lamp, and a straight fluorescent lamp.

1. Turn on the incandescent lamp and observe it with the spectrometer by looking through the circular opening while the slit is aligned parallel to the filament. Describe the spectrum you see.

2. Turn on the mercury-vapor lamp and observe it with the spectrometer. Align the slit with the long dimension of the tube. Describe the spectrum you see. How does it differ from that of the incandescent lamp?

3. Turn on the fluorescent lamp and observe it with the spectrometer. Describe the spectrum. How does it differ from the previous two?

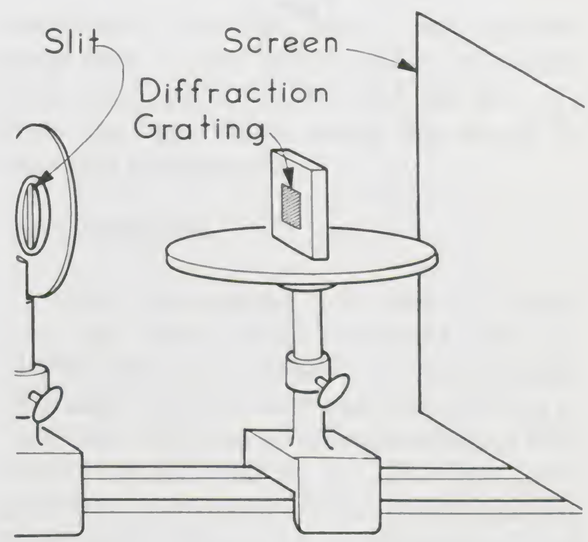


Figure 8.

Part III

You will now use the straight filament lamp and DC-power supply from Experiment A-1 to observe changes in the spectrum as voltage is changed. To observe the spectrum you will use a prism.

1. Set the voltage at 100 V. Hold the prism near the straight filament lamp as shown in Figure 9. Turn the prism as you look into it until you see the spectrum of the filament. Do all colors of the spectrum (red, orange, yellow, green, blue, and violet) seem equally bright?

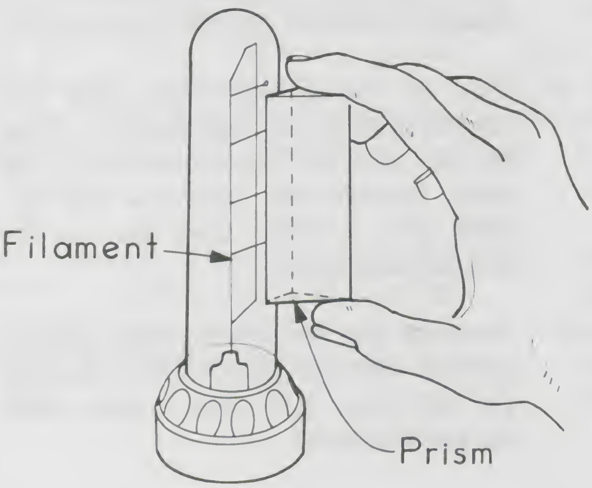


Figure 9.

2. Observe the spectrum as you decrease the voltage to the lamp. How does the brightness of the colors change as the voltage is decreased?

3. Decrease the voltage until the filament is just glowing. How is the light now distributed among the colors of the spectrum?

4. Increase the voltage as you observe the spectrum. How do you think the color of the filament as observed in Experiment A-1 is related to the distribution of light in the spectrum?

5. How do you think the distribution of light in the spectrum is related to temperature?

EMISSION SPECTRA

The spectrum emitted by a heated solid, such as the filament of an incandescent lamp, is called a *continuous spectrum.* The colors of this spectrum merge smoothly into one another from the violet to the red with no gaps. The colors seen in this kind of spectrum are known as the spectral colors: red, orange, yellow, green, blue, violet, and the various intermediate shades leading from one hue to the next. The spectrum of sunlight is quite similar to that of a hot solid. By analyzing sunlight with a prism in the seventeenth century, Sir Isaac Newton was the first to prove that white light is composed of a mixture of the spectral colors.

As you saw in Experiment A-2, the light from mercury vapor* has a spectrum which is very different from a continuous spectrum. Only a few widely separated spectral colors were present. You may have seen a violet *line,* a blue *line,* a green *line,* a yellow *line,* and perhaps a dim red one. This type of spectrum is called a *line spectrum.* The lines you saw in the spectroscope were really colored images of the slit.

Many kinds of mercury vapor lamps are important in illumination. Radiation from such lamps typically consists of a combination of line and continuous spectra.

It would be a simple matter to replace the mercury vapor tube with tubes containing other gases in order to show that other gases emit line spectra. Different gases will give different sets of colored lines. You did not view any others since this module is primarily concerned with incandescent lamps.

The fluorescent lamp produces a spectrum that is a combination of the two spectral types: you can see a continuous spectrum, but a few lines of color stand out more brightly than the background. The bright lines come from mercury vapor inside the tube (you may have noticed that they fell at exactly the same positions in the spectrum as those of mercury vapor), while the continuous spectrum comes from *phosphors* coating the inside wall of the tube.

All three of the light sources appear white or nearly white to the unaided eye. But the light from each is quite different, being made up of different combinations of colors. The eye, by itself, cannot analyze light, but rather gives only one color sensation from each point, even though many spectral colors may be coming from that point. Sometimes a mixture of spectral colors gives rise to a color sensation that is not in any spectrum, such as brown, purple, etc. Such colors are called *non-spectral colors.* We therefore should not be surprised when an object appears differently colored when viewed under incandescent lighting, fluorescent lighting, mercury-vapor lighting, or sunlight. In these separate cases, the object has different mixtures of spectral colors falling on it, and thus different mixtures being reflected.

Question 5. The light from a mercury-vapor lamp looks bluish, and its spectrum features blue-violet, green, yellow, and red lines. To make the light whiter, which line should be increased in intensity?

WAVELENGTH

The spectrometer you used in Experiment A-2 has a small numerical scale displayed under the position of the spectrum. The marks on this scale run from 400 up to 700. The violet end of the spectrum is at 400, while the red end is at 700. This scale provides a convenient method for specifying the spectral colors, since a given spectral color will always fall above the same number on the scale. Therefore, when we refer to some spectral color, instead of trying to describe an exact shade we can give a number. For example, instead of talking about the blue-violet line of mercury vapor, we could say the 440 line.

Actually, this scale of numbers represents what is known as wavelength and has units of length. The scale runs from 400 to 700 nanometers (nm). The name *wavelength* stems from the fact that light behaves as if it were a wave. One experiment which helps

*Although a technical distinction exists between a *vapor* and a *gas*, the two terms will be used interchangeably in this module.

to confirm this is the breaking up of light into a spectrum by the diffraction grating. This grating is a pattern of tiny transparent and opaque strips something like the slats and spaces of a venetian blind. To explain this action of the grating, it is necessary to assume that light has wave properties. However, we will not attempt such an explanation here because it would take us too far afield from the incandescent.

The concept of wavelength can be visualized with the aid of Figure 10.

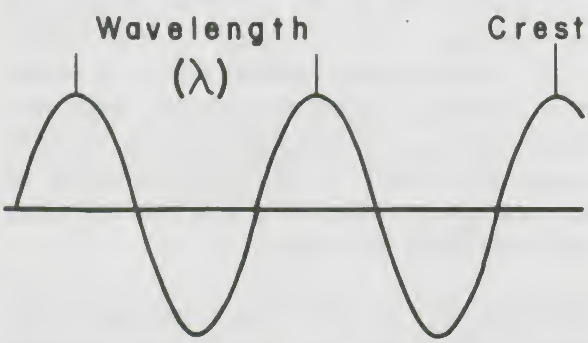


Figure 10.

As indicated in the figure, wavelength can be defined as the distance between two successive crests of a wave. The Greek letter *lambda* (λ) is used to represent this quantity.

Question 6. As read from your spectrometer what is the wavelength of the blue-violet line in the mercury-vapor spectrum?

Question 7. Is the wavelength of red light larger or smaller than an ant? How much so?

The heat radiated by a hot solid is identical to the light radiated. Both radiations exhibit wave properties. The only physical difference is that the heat radiation behaves as waves with considerably longer wavelengths. Heat radiation is called *infrared* radiation.

If we describe the spectra of various light sources in terms of wavelength instead of colors, we say that a hot solid emits energy at all visible wavelengths and also at infrared wavelengths.

The line spectrum of a gas is due to the emission of only certain definite wavelengths.

THE ELECTROMAGNETIC SPECTRUM

Light and infrared waves are a part of a larger class of waves called *electromagnetic waves.* The name stems from the fact that the waves consist of changing electric and magnetic fields. All electromagnetic waves travel at the speed of light (almost 3×10^8 m/s) in empty space. The wavelengths of electromagnetic waves range from very small to very large. The set of all such possible wavelengths is called the *electromagnetic spectrum.* Different portions of the electromagnetic spectrum have been given different names for convenient reference.

Figure 11 shows a diagram of the electromagnetic spectrum and an enlargement of the visible portion, which we call light. The wavelengths are in *nanometers*: 1 nm = 10^{-9} m.

Question 8. What would be the names given to electromagnetic waves with the following wavelengths: 1 cm (10^7 nm); 2000 nm; 600 nm; 1 nm?

WAVELENGTH AND THE EYE

Those wavelengths of the electromagnetic spectrum to which the human eye responds are called visible light. The eye responds differently to different visible wavelengths. That fact should already be apparent to you from Figure 11, which shows that different wavelengths appear to be different spectral colors. However, even if we disregard the color sensations, the eye responds differently to different wavelengths. The same amount of energy per unit time (power) radiated at different wavelengths gives rise to different sensations of brightness. We shall study this effect in more detail later, only noting here that the spectrum of a light source determines not only its color, but also how bright it appears to be. As you saw in Experiment A-1, brightness also increases with total power.

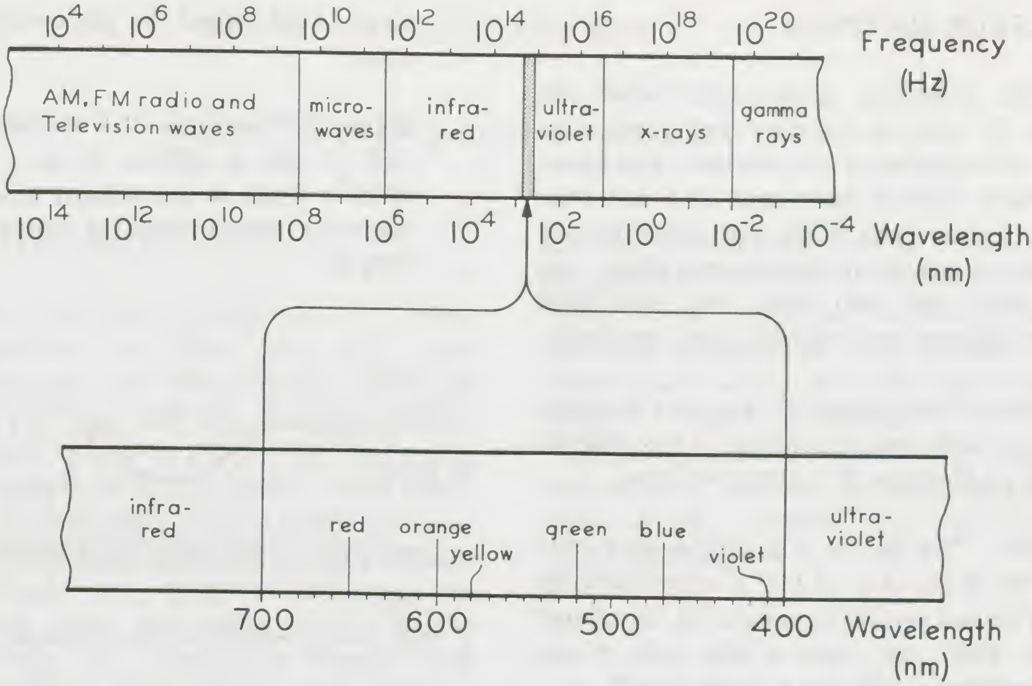


Figure 11. The Electromagnetic Spectrum. The boundaries between the radiations with different names are not at all sharp, and borderline wavelengths may be called either one or the other. Note that, in the top scale, going to the left, each mark represents a wavelength ten times greater than the preceding mark.

SUMMARY

The incandescent lamp works by heating a small wire called a *filament* to a very high temperature with an electric current. The filament, like any hot solid, emits energy as *electromagnetic waves* in a *continuous spectrum.* At the temperature of the filament, most of the power emitted is in the *infrared* and *visible light* regions of the electromagnetic spectrum. Our skin detects the infrared waves as heat, and our eyes detect the light.

Other light sources such as *mercury-vapor lamps* produce *line spectra.* In these sources light is not produced by a hot solid.

If the power radiated at each visible wavelength (all spectral colors) is approximately the same, one gets the sensation of "white," as from the incandescent lamp. However, any light source whose spectrum spans the visible region (some light in the red, green, and blue) will generally appear to be white. The different shades of "white light" from different types of lamps arise from the different spectra emitted.

The *brightness* of a light source depends both on the total power emitted and on how that power is distributed in the spectrum.

GOALS FOR SECTION B

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal. Answers appear immediately after these goals.

1. *Goal:* Understand the absolute temperature scale and its relation to the pressure of an ideal gas at constant volume.

   Item: The gas in a gas-filled light bulb goes from 25°C to 110°C when the lamp is turned on. If the gas can be considered an ideal gas, what is the ratio of gas pressures at the two temperatures?

2. *Goal:* Know how the electrical resistance of a conductor varies with temperature.

   Item: The resistance of a piece of copper wire is 2×10^{-2} Ω at 0°C and its temperature coefficient of resistance is 4×10^{-3}/C°. What is the wire's resistance at 125°C?

3. *Goal:* Understand the definitions of blackbodies and graybodies.

   Item: The three graphs in Figure 12 show percent of incident electromagnetic radiation absorbed versus wavelength. Which graph(s) show the results for blackbodies or graybodies?

4. *Goal:* Know the definitions of absorptance, emissivity, irradiance, and radiant exitance (formerly emittance).

   Item: A graybody with an emissivity of 0.6 and a surface area of 1.5×10^{-3} m^2, emits 150 W of radiant power. What is the radiant exitance of a blackbody at the same temperature as the graybody?

5. *Goal:* Understand the Stefan-Boltzmann Law.

   Item: A blackbody at a temperature of 300 K has a radiant exitance of 459 W/m^2. What is the radiant exitance of the same object when the temperature is 900 K?

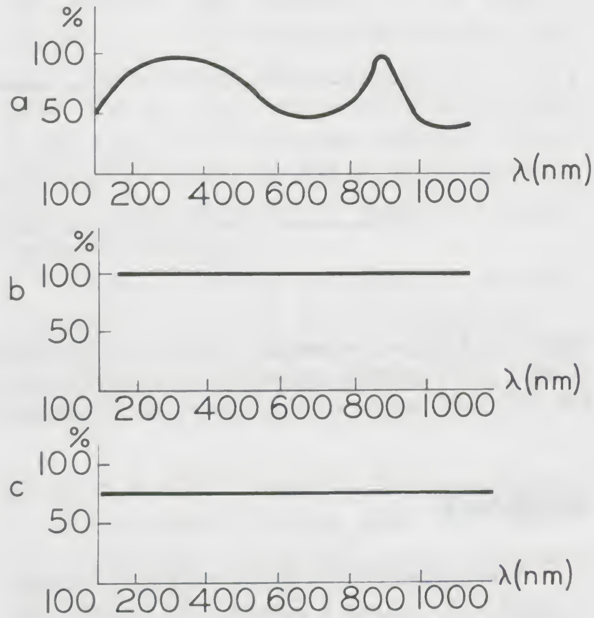


Figure 12.

Answers to Items Accompanying the Preceding Goals

1. The pressure at the higher temperature is 1.29 times the pressure at the lower temperature.

2. 3×10^{-2} Ω

3. a. neither blackbody nor graybody.
   b. blackbody.
   c. graybody.

4. 1.67×10^5 W/m^2

5. 37,000 W/m^2

SECTION B

An Empirical Approach

GAS-FILLED BULBS

For lamp sizes of greater than 25 watts, manufacturers have found that inert gases inside the glass bulb are necessary. Although this gas prolongs bulb life, its presence produces some unwanted effects. The gas next to the filament is heated and tends to rise to the top of the bulb and circulate down the sides. These gas motions, called *convection currents,* transfer heat energy from the filament to the bulb wall where the energy is lost to the surroundings. As a result, the filament does not stay quite so hot or glow quite so brightly as it would without the gas. As much as 20% of the power supplied to the lamp can be lost through gas convection.

The average gas temperature may increase to 100°C or more above room temperature, which is a significant change because this greatly increases the pressure of the gas. A graph of pressure versus temperature (in degrees celsius) is nearly a straight line, but the line does not pass through the origin (see Figure 13). That is, when the temperature is 0°C, the pressure is *not* zero. If we extend this graph backward until it does cross the temperature axis, we can find the temperature which would correspond to zero pressure. If this new temperature were the zero of our temperature scale, then the gas would be simply a direct proportion. The resulting temperature scale is important in science and technology. You can find an approximate value of this "absolute zero" by doing the next experiment.

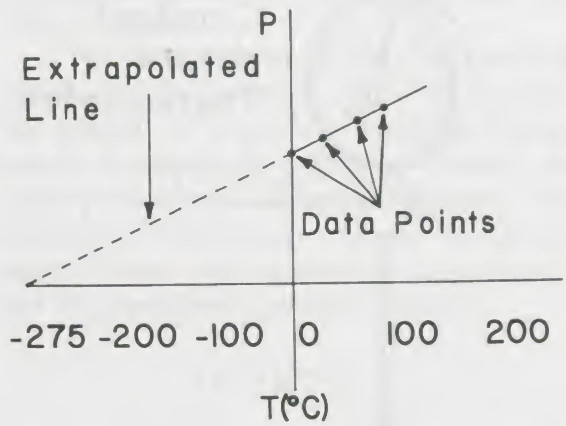


Figure 13.

EXPERIMENT B-1. Determining the Value of Absolute Zero (Optional)

A type of thermometer which can be used for low temperature measurements (below the freezing point of most liquids) is the constant-volume thermometer pictured in Figure 14. This thermometer makes use of the fact that for a constant volume, the pressure of a gas varies linearly with its temperature. The purpose of this experiment is to determine the temperature at which the pressure of the gas would become zero.

In order to determine the temperature at which the pressure would become zero, measure the pressure corresponding to three different temperatures.

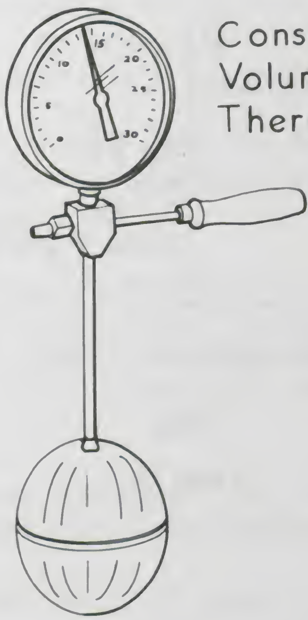


Figure 14.

1. With the constant-volume thermometer and the mercury thermometer on the lab table and at the same temperature as the air, record on the work sheet the pressure from the constant-volume thermometer and the temperature in °C from the mercury thermometer. Do not touch the bulb of either thermometer as you make these measurements, because this would change the readings slightly.

2. Submerge the bulb of the constant-volume thermometer completely into ice water. Record the pressure on the work sheet. Measure the temperature of the ice water with the mercury thermometer and record it on the work sheet.

3. Submerge the bulbs of both thermometers in boiling water. Measure and record the pressure and the temperature.

4. Graph the three points, putting pressure on the vertical axis and temperature on the horizontal axis. Draw a straight line as nearly as possible through these data points. Extend this line until it crosses the temperature axis.

5. The temperature at which the pressure of a gas would become zero, called *absolute zero,* is the point at which your line intersects the temperature axis. What is your value for absolute zero? This value is called an *extrapolated* value. It is the point where the gas pressure would become zero, if the substance did not liquify or solidify first.

6. Can you write an equation relating pressure and temperature as depicted by your graph?

PRESSURE AND ABSOLUTE TEMPERATURE

As one can infer from Experiment B-1, if we start with a gas at some pressure and at 0°C, the pressure increases by a particular fraction of the original pressure for every degree increase in temperature. The pressure decreases by the same fraction of the original pressure for every decrease in temperature of one degree. Careful experiments have shown this fraction to be about 1/273. Actually this law is only approximately obeyed, but the interesting point is that all gases more or less follow the law. What would happen if the temperature is lowered by 273°C? If the gas continued to behave the way it did near 0°C, the pressure would decrease by 273/273 of the original pressure. In other words, the pressure would fall to zero at −273°C. Such behavior defines what scientists call an *ideal gas.* Of course, an actual gas would condense into a liquid before reaching such a low temperature, but that fact does not lessen the significance of the −273°C temperature. This temperature is an absolute minimum temperature. In fact, a new temperature scale, named the *absolute* or *kelvin* temperature scale, is used to express this fact. The zero of the kelvin scale corresponds to −273°C, and kelvins (K) are the same size as Celsius degrees. Therefore, one may convert a Celsius temperature to an absolute temperature by just adding 273. In equation form, if we let T_C be the Celsius temperature, absolute temperature, T_K, is given by

$$T_K = T_C + 273 \qquad (1)$$

This temperature scale allows us to write a simple relation between the pressure and temperature of a gas at constant volume by moving the origin of the graph you constructed in Experiment B-1 to −273°C. Then the pressure, p, is proportional to the absolute temperature T_K

$$p \propto T_K$$

For example, a typical light bulb has inside a gas pressure of ¾ of an atmosphere (1 atm = 14.7 lb/in^2) at 27°C. When we turn the bulb on, the temperature increases to 127°C. To find the pressure of the gas while the bulb is on, first convert the initial and final temperature to the kelvin scale: $T_1 = 27 + 273 = 300$ K, $T_2 = 127 + 273 = 400$ K. From the proportionality, $p \propto T_K$, we may form the proportion, $p_2/p_1 = T_2/T_1$, and solve for p_2. Thus $p_2 = p_1(T_2/T_1) = 3/4 \times 4/3 = 1$ atm.

This example illustrates why manufacturers fill lamps to a pressure which is less than atmospheric pressure. Then, during operation the pressure inside will be about the same as the pressure exerted by the atmosphere. An operating pressure higher than one atmosphere inside the bulb could cause the bulb to explode.

The pressure referred to in the gas law is called *absolute pressure.* When we measure the pressure of a gas in a container, we are usually measuring the *difference* between the inside pressure and atmospheric pressure. This difference is called *gauge pressure.* To connect gauge pressure (P_g) to absolute pressure (P_a), add the atmospheric pressure. That is

$$P_a = P_g + P_{atm}$$

Problem 1. An automobile tire which is inflated with air to 30 pounds per square inch (30 psi gauge) when the temperature is 27°C becomes heated as the automobile is driven. In the process the temperature of the air in the tire rises to 77°C. If the volume of the tire does not change, what would be the gauge pressure in the tire at this higher temperature?

Problem 2. A constant volume gas thermometer is used to measure very low temperatures. It consists of a container filled with helium gas at a pressure of 1 atm and temperature of 27°C. The thermometer is placed in liquid oxygen to measure its temperature. If the pressure then reads 0.3 atm, what is the temperature of the liquid oxygen in degrees Celsius?

EXPERIMENT B-2. The Relationship of Resistance to Temperature for a Conductor

Another quantity that changes as the incandescent lamp heats up is the electrical resistance of the filament. You discovered this change yourself in Experiment A-1, at least qualitatively. You can investigate this effect by doing this experiment.

The purpose of this experiment is to determine the relationship between the resistance and temperature of a conductor. You will measure the resistance of a filament at various temperatures, using an ohmmeter. To do this, place a mixture of crushed ice and water in a beaker and place it on the ring stand. Stir this mixture frequently while preparing the rest of the apparatus. You need an exposed, incandescent lamp filament mounted in a lamp socket. Connect the wires from the lamp socket to the ohmmeter. Your set-up should now look similar to that shown in Figure 15.

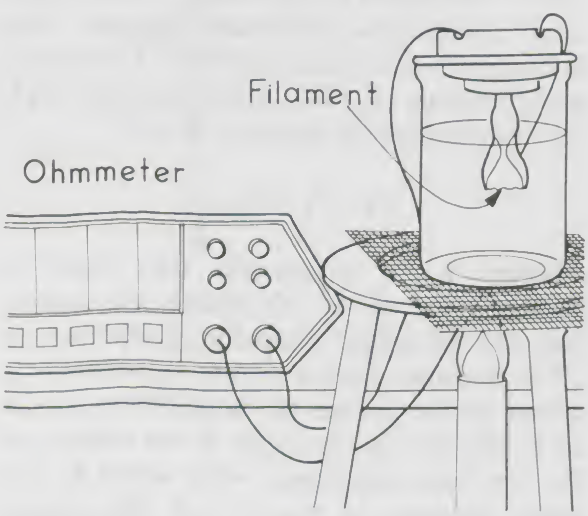


Figure 15.

Invert the lamp socket and insert the filament in the beaker so that the filament is completely immersed in the ice water.

Using the ohmmeter, measure the resistance of the filament. Insert a thermometer into the water and measure the temperature. Record these measurements on the work sheet.

Light the bunsen burner and slowly heat the ice water until the ice has melted and the water temperature has increased to about 10°C. Remove the bunsen burner from under the beaker and allow one minute for the temperature to stabilize. Measure and record the resistance and the temperature.

Continue taking measurements at approximately ten degree intervals, until the water boils. Record these measurements on the work sheets.

Plot a graph of resistance (in ohms) on the vertical axis and the temperature (in degrees celsius) on the horizontal axis.

1. Do the points on your graph appear to lie in a straight line?

2. Draw a straight line through your data points. The best line is one for which approximately as many data points are above the line as there are below.

3. Find the slope (rise ÷ run) of the line. What are the units of the slope?

4. What is the value of the vertical intercept? The vertical intercept is the point where the graph crosses the vertical axis, or where $T = 0$°C.

5. What are the vertical-intercept units?

6. If you divide the slope of your graph by the vertical intercept, you get a constant which describes the dependence of the resistance of the given material on its temperature. This constant is called the *temperature coefficient of resistance.* (The temperature coefficient of resistance is defined as the fractional change in resistance per Celsius degree change in temperature measured at 0°C.) Calculate the temperature coefficient of resistance for your tungsten filament.

7. What are the units of the temperature coefficient of resistance?

8. Can you write an equation to describe the temperature dependence of resistance for your filament?

TEMPERATURE COEFFICIENT OF RESISTANCE

You found in Experiment B-2 that the resistance of tungsten increased with increasing temperature and that, at least approximately, this increase is *linear.* That is, the graph of resistance versus temperature is a straight line. The linear dependence of resistance on temperature for a conductor can be expressed by the equation

$$R = R_o + (\text{slope}) \times T$$

Where R_o is the vertical intercept of a graph of R versus T. If this equation is written using the *temperature coefficient of resistance,* α_T, it has the form

$$R = R_o(1 + \alpha_T T) \qquad (2)$$

(Relationship of resistance to temperature for a conductor)

where R is the resistance at celsius temperature T and R_o is the resistance at 0°C.

The variation of resistance with temperature is quite important for incandescent lamps, since the filament temperature may go from about 20°C when turned off to over 2000°C when the lamp is on. Over this range, the resistance of the filament may actually increase by as much as a factor of 20.

Example Problem. Platinum is a metal often used in low temperature thermometers. It has a temperature coefficient of resistance α_T of 3.6×10^{-3}/C°. Calculate the resistance of a platinum resistance thermometer at $T = -180$°C, if its resistance is 0.5 Ω at 0°C.*

Solution. Given in the problem are:

$$\alpha_T = 3.6 \times 10^{-3}/\text{C}^\circ$$
$$T = -180^\circ\text{C and}$$
$$R_o = 0.5\ \Omega$$

*In this module, °C is used to denote a temperature point, whereas C° is used to denote a change in temperature or difference between two temperatures. The first is a *point* on the scale, the second a *distance* along the scale. The notation, /C° means "per celsius degree," or 1/C°.

Substitution into Equation (2) gives

$$R = 0.5\ \Omega\ [1 + 3.6 \times 10^{-3} \times (-180)]$$

Multiplying inside the parentheses gives

$$R = 0.5\ \Omega\ (1 - 0.65)$$
$$= 0.5\ \Omega \times 0.35$$

Our value for the resistance at −180°C is then

$$R = 0.18\ \Omega$$

Problem 3. The temperature coefficient of resistance for copper is 4.0×10^{-3}/C°. Calculate the resistance of a bar of copper at 100°C, if its resistance at 0°C is 0.10 Ω.

Example Problem. The nickel lead-in wires in a typical light bulb have a resistance of about 0.08 Ω at 0°C. When the lamp is turned on, the nickel wire reaches a temperature of about 200°C, and its resistance changes to 0.18 Ω. From these data, calculate the temperature coefficient of resistance for nickel.

Solution. You are given in the problem that $R_o = 0.08\ \Omega$, $R = 0.18\ \Omega$ and $T = 200$°C. Equation (2) must now be rearranged to solve for α_T. As a start the equation may be divided by R_o to give

$$R/R_o = 1 + \alpha_T T$$

Subtracting 1 from each side yields

$$R/R_o - 1 = \alpha_T T$$

Now dividing by T and reversing the sides give

$$\alpha_T = (R/R_o - 1)/T$$

Substituting given values into this gives

$$\alpha_T = [(0.18\Omega/0.08\Omega) - 1]/200\text{C}^\circ$$

Dividing the resistances and cancelling ohms gives

$$\alpha_T = (2.25 - 1)/200\text{C}^\circ$$

Finishing the arithmetic and expressing the answer in powers of ten gives

$$\alpha_T = 6.3 \times 10^{-3}/C°$$

Problem 4. The resistance of a certain metal is 100 Ω at 0°C. If its resistance rises to 122 Ω when its temperature is raised to 60°C, what is the temperature coefficient of resistance for this material?

Question 9. When a toaster is first turned on, the resistance of the heating element is much less than it is after it heats up. Describe what happens to the current in the toaster from the instant it is first turned on until the heating element reaches its red-hot state.

RELATIVE RESISTANCE OF TUNGSTEN

The change in resistance for a substance like tungsten can be displayed in a graph of *relative resistance,* which is the resistance at a given temperature, R_T, divided by the resistance at room temperature, R_{300}. Figure 16 shows this graph using absolute temperature.

The graph in Figure 16 can be used to find the resistance of a piece of tungsten at a high temperature if the resistance of that piece at 300 K (room temperature) and the high temperature are both known. However, in practice it is often easier to measure the electrical resistance of a substance at a high temperature than it is to measure the high temperature. Therefore a graph like Figure 16 can be a convenient tool for calculating high temperatures by measuring resistances.

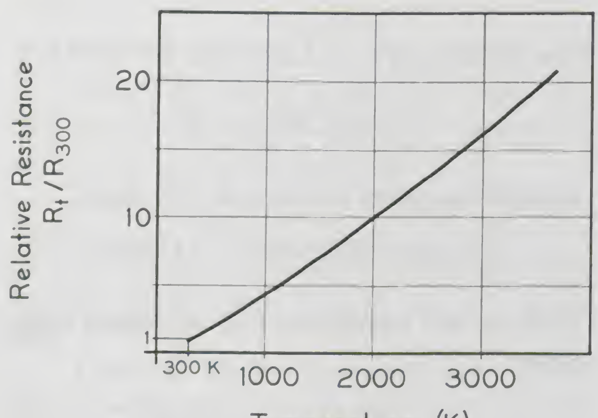


Figure 16.

Example Problem. A piece of tungsten has an electrical resistance of 2 Ω at room temperature. When it is heated to a high temperature, its resistance is measured to be 30 Ω. What is the final temperature of the sample?

Solution. You are given that $R_{300} = 2\ \Omega$ and $R_T = 30\ \Omega$. First the ratio is calculated

$$R_T/R_{300} = 30/2 = 15$$

Knowing that the relative resistance is 15 allows you to consult the graph in Figure 16 to read the temperature, $T = 2800$ K.

Problem 5. A piece of tungsten has a resistance of 2 Ω at room temperature and a resistance of 15 Ω at a much higher temperature. What is the higher temperature?

Problem 6. What is the temperature of tungsten when its relative resistance is ten?

Problem 7. If the melting point of tungsten is 3380°C, what is the largest value of relative resistance a solid piece of tungsten can have?

RADIANT POWER AND TEMPERATURE

The power radiated by a hot solid, like a tungsten filament, depends on its temperature. Your observations in Experiment A-1 indicated that more power is radiated by a solid when its temperature is higher. Now that we have a method for measuring high temperatures, it is possible to investigate more precisely the relation between the temperature of a hot solid and the total power it emits as thermal (heat) radiation (*radiant power*). The filament of a 25-W incandescent lamp is a good experimental object, because over 90 per cent of the electrical power put into it is emitted as thermal radiation. We can make the approximation that the power radiated is the same as the measured electrical power input. Higher wattage, gas-filled lamps have significant gas convection energy losses which would make this approximation far less valid.

EXPERIMENT B-3. Relationship of Radiated Power to Temperature

In order to investigate more carefully the relationship between radiated power and the temperature of a solid, you will measure the input power and the temperature of an incandescent lamp filament.

Connect the 25-W lamp and the meters to a DC power supply as shown in Figure 17. The ammeter is used to measure the current through the filament, and the applied voltage is measured with the voltmeter.

Adjust the power supply to produce a current of 0.100 amperes (A) through the filament. Record both the current I and the voltage V in the table on the work sheet. Increase the current by 0.020 A and record the current in amperes and the voltage in volts. Continue in steps of 0.020 A until a maximum of 0.260 A is reached. Record each current and voltage in the table.

In order to find the temperature of the filament you will need to know its resistance at 0°C. To measure this resistance, it is necessary to break the bulb and remove the filament from the envelope. Because of the shattering of the glass envelope, it is dangerous to break the bulb without taking some precautions. Place masking tape around the bulb close to the base. Make sure the tape is sticking firmly all the way around. Next place the metal base of the bulb in a vise and tighten until you hear the bulb break. The bulb should break neatly around the base. Carefully lift away the glass bulb from the base without letting it touch the filament. You then have the base with the filament intact.

1. Carefully place the base in the lamp socket. With the filament immersed in ice water measure its resistance as in Experiment B-2. Record the resistance at the temperature of the ice water (R_o) on the work sheet.

The electrical power input, in watts, to the filament is calculated by multiplying the current by the voltage:

$$P = IV$$

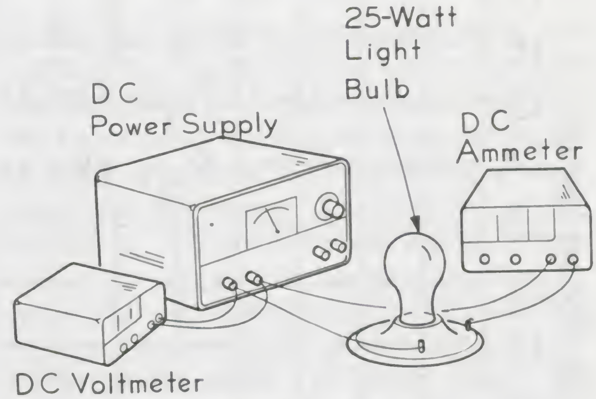


Figure 17.

Calculate and record the power input for each current and voltage setting.

In order to find the temperature of the filament, the relative resistance of the filament must be calculated. The resistance, in ohms, is found by dividing the voltage by the current (Ohm's Law):

$$R = V/I$$

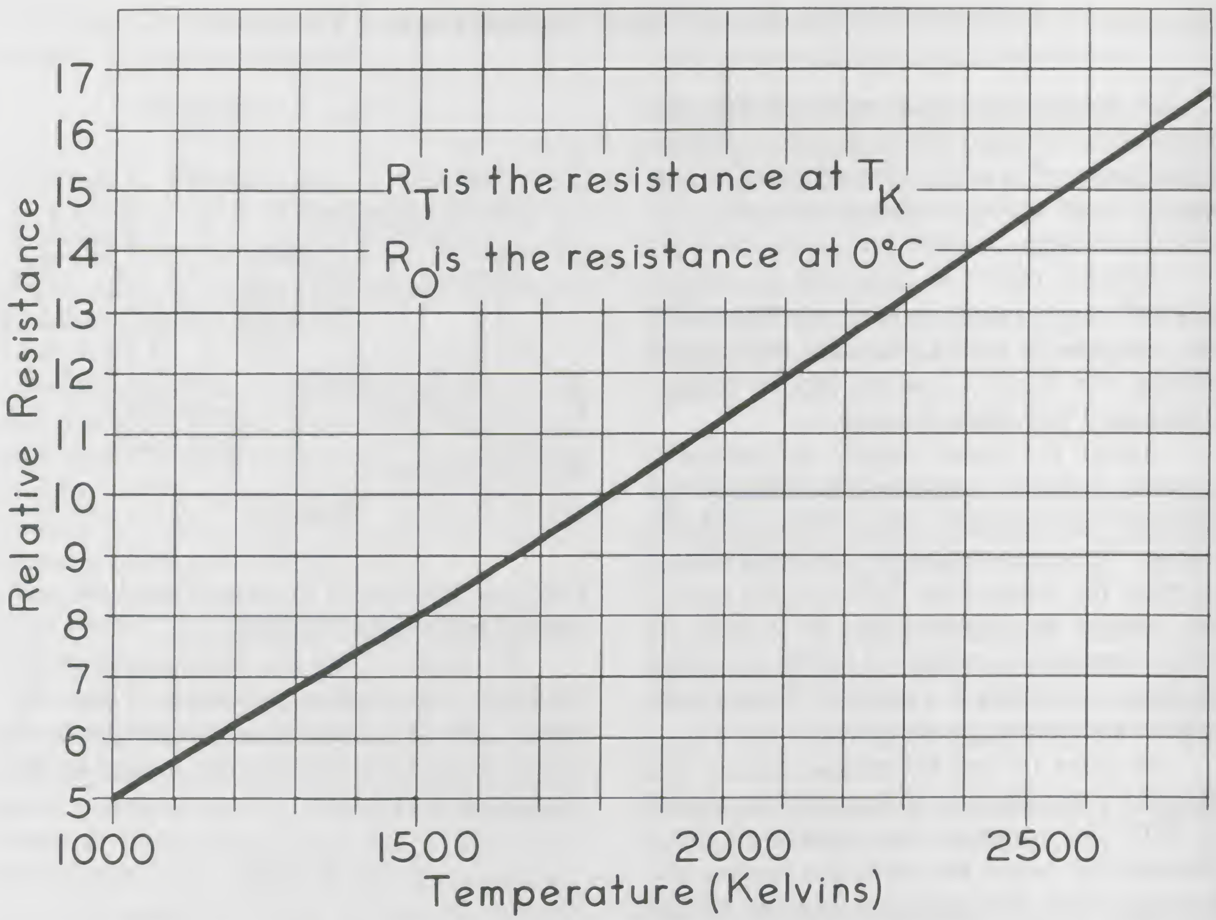
Calculate and record the resistance for each current and voltage.

The relative resistance R/R_o, is the ratio of the resistance at some temperature T to the resistance at 0°C. Calculate and record the relative resistance for each current reading.

The temperature in each of these cases can be found from a graph of relative resistance and temperature. Use Figure 18 to determine the operating temperatures of the filament. Record these temperatures, in kelvins. You should notice that Figure 18 differs from Figure 16 in that the reference temperature, where R equals R_o, in Figure 18 is 0°C (273 K) while for Figure 16 it is room temperature (300 K).

The task which now needs to be approached is that of finding the relationship between the temperature of the radiating body and the power radiated. If the graph of power versus temperature is a straight line, the relationship is said to be *linear.*

2. Plot a graph of input power (P) on the vertical axis and temperature (T_K) on



Resistance properties of tungsten

Figure 18.

the horizontal axis. For a 25-W bulb, you can safely assume that the radiated power is the same as the input power. Is this graph a straight line?

If your graph of P and T_K were a straight line, the linear relationship between power and temperature would be written as

$$P = mT_K + P_o$$

where m is the slope of the line and P_o is the radiated power at $T_K = 0$ K. It is logical that no power is radiated at 0 K; therefore, $P_o = 0$. But your graph is not a straight line, therefore the relationship is more complicated. It is, however, usually possible to analyze data to find some equation which relates two quantities. If a linear relationship has a graph which passes through the origin, then it is a special case of a more general mathematical relationship, which we can write as

$$P = mT_K^n$$

where n is an unknown number. This equation is just one possibility. You can now analyze your data to see if the relationship is of this type because it is relatively easy to do so. We should emphasize that there is no guarantee that this equation is valid for *any* value of n. If not, then a still more complicated equation would be tried, and so on, until one is found which works. To see whether this relationship does describe your data, and to find the value of n, you can proceed in two different ways. The first way is more straightforward, but it involves tedious calculations. The second way, based on the properties of logarithms, avoids these calculations, thus it is a faster method.

METHOD I. Trial and Error

By looking at your graph of P versus T_K, you can learn something about n. n is positive if the radiated power increases as temperature increases. That is, n is positive if the slope of the graph is positive. n is negative if the power decreases as temperature increases. That is, n is negative if the slope of the curve is negative. If n is larger than one, a small change in temperature produces a large change in power. Assume, for simplicity, that n is a whole number (integer).

You can check to see if a particular value of n works by making a new graph of P on the vertical axis and T_K^n on the horizontal axis. If this graph is a straight line through the origin,

$$P = mT_K^n$$

This is true because the equation says that the power is proportional to the temperature raised to the exponent n. That is, the relationship between P and T_K^n is linear.

1. Make a new table of input power and corresponding filament temperatures. Extend this table to include values of T_K^n for $n = 2, 3, 4$, etc. An electronic calculator will be useful for these calculations.

2. Plot P on the vertical axis and T_K^n on the horizontal axis for $n = 2, 3$, and 4. Is any one of these graphs linear? Does it pass through the origin? If so, you have found a value for the exponent n.

3. Write the relationship between radiated power and filament temperature as determined from your graphs. Remember that you are assuming that radiated power is the same as input power.

METHOD II. Log-Log Plotting (Optional)

Data analysis by graphical methods often may be simplified by using the properties of logarithms. Notice what happens when you take the logarithms of both sides of

$$P = mT_K^n$$
$$\log P = \log m + n \log T_K$$

If the first of these equations is valid, the graph of $\log P$ against $\log T_K$ will be a straight line with slope n. Simply plotting $\log P$ and $\log T_K$ not only allows a quick determination of whether or not the first equation is valid, but it also allows easy determination of the value of n. You can make such a graph in two ways. One way is to use ordinary graph paper and a table of logarithms to plot $\log P$ and $\log T_K$. An equivalent, but simpler way is to use *log-log graph paper.* On such paper, the distance along either axis to a particular coordinate number is proportional to the logarithms of that number. Instead of having to look up the logarithm and measure a distance proportional to it, you only need to plot the number according to the scale indicated on the paper. The log-log paper automatically computes the logarithm for you. The number of times the logarithmic scale (from 1 to 10) is repeated on each axis is designated as the number of *cycles.*

1. Using the two-cycle log-log graph paper, plot P on the vertical axis and T_K on the horizontal axis. Your data points should appear to lie along a straight line. Draw the straight line which best fits your data.

   In order to determine n, you need to find the slope of the line. Because the scales on the graph are logarithmic, you cannot measure the rise and the run by using the scales. You must use a ruler to find the slope.

2. Pick two points on the straight line. With a ruler, measure the rise and the run of the line between these two points. Calculate the slope by dividing the rise by the run and round it off to the nearest whole number.

3. Write the relationship between radiated power and filament temperature as determined from your graph. Remember that you are assuming that the radiated power is the same as the input power. You are also assuming that at $T_K = 0$, $P = 0$.

BLACKBODIES AND GRAYBODIES

In Experiment B-3, you discovered that the power radiated from a tungsten filament is roughly proportional to its absolute temperature raised to the fourth power. This relation between radiated power and temperature is approximately true for most solids. To simplify the calculations, scientists define idealized solids which obey exactly this fourth-power relationship. They are called *blackbodies* or *graybodies.* Then actual solids can be considered as objects which deviate from the ideal.

Question 10. How much does the power radiated from the filament of an incandescent lamp change when the absolute temperature of the filament is doubled?

You have already seen that white light sources emit energy all across the visible spectrum. The "whitest" light comes from sources that emit about equal power at all visible wavelengths. Most visible objects are seen by reflected light. If we assume that a white light source illuminates an object that reflects all of the power at all wavelengths, the illuminated object also appears to be a white light source, since the same light is coming from it as from the source. If the object reflects all wavelengths equally well, but it does not reflect all of the incident power (say, it reflects 50% and absorbs 50% of the power at all wavelengths), it still appears to be neutral in color, but it is not so bright as the source. In other words, it appears gray. If the object absorbs all power at all wavelengths, reflecting nothing, it appears to be black. Such objects which reflect and absorb all wavelengths are the ones which exactly obey the fourth-power dependence on temperature for the emission of thermal radiation. Of course, real objects are never exactly like these perfect reflectors.

RADIANT EXITANCE

We may generalize these ideas and say that a *blackbody,* by definition, absorbs *all* electromagnetic radiation of any wavelength, visible or not, that falls on it. A *graybody*, by definition, absorbs some fixed fraction of all wavelengths. This discussion may be made more precise by making certain definitions:

The radiation falling upon some object is called *incident irradiance,* and is defined as

$$E_{\mathrm{inc}} = P_{\mathrm{inc}}/a_{\mathrm{s}} \tag{3}$$

where P_{inc} is the total radiant power falling on the object and a_{s} is its *surface area.*

The radiation absorbed by an object is called *absorbed irradiance,* and is defined as

$$E_{\mathrm{abs}} = P_{\mathrm{abs}}/a_{\mathrm{s}} \tag{4}$$

where P_{abs} is the total radiant power absorbed by the object and a_{s} is its surface area.

A measure of the radiation given off by some object is called *radiant exitance,* and is defined as

$$M = P_{\mathrm{R}}/a_{\mathrm{s}} \tag{5}$$

where P_{R} is the radiant power emitted by the object and a_{s} is its surface area. (Radiant exitance was formerly called *radiant emittance,* and you may still find it listed that way in many handbooks.)

The units for all three of these quantities are watts per square meter ((W/m^2).

Sometimes we wish to know what fraction of the radiation striking a surface is absorbed by the surface of an object. For this we use the term *absorptance,* α, defined by

$$\alpha = E_{\mathrm{abs}}/E_{\mathrm{inc}} \tag{6}$$

It is often useful to know how the radiation emitted by some object compares with that radiated from a blackbody which is at the same temperature. For this we use the term *emissivity,* defined by

$$\epsilon = M/M_{\mathrm{bb}} \tag{7}$$

where M is the radiant exitance of the object and M_{bb} is the radiant exitance of a blackbody at the same temperature.

In general, both the absorptance α and the emissivity ϵ depend on the wavelength of

the radiation being absorbed or emitted. For blackbodies and graybodies, α and ϵ do *not* depend on the wavelength, but are the same for *all* wavelengths. Both α and ϵ are just numbers, with no units, and both may range from zero to one.

Example Problem. Irradiance of 100 W/m^2 is incident on an object; 70 W/m^2 is absorbed. What is the absorptance of the sample?

Solution. Given are

$$E_{abs} = 70 \text{ W/m}^2$$

and

$$E_{inc} = 100 \text{ W/m}^2$$

Substituting into Equation (6) gives

$$\alpha = (70 \text{ W/m}^2)/(100 \text{ W/m}^2)$$

and

$$\alpha = 0.70$$

Example Problem. A dark block of metal at a temperature of 300 K has a radiant exitance of 367 W/m^2. A blackbody at this same temperature has a radiant exitance of 459 W/m^2. What is the emissivity of the metal block?

Solution. Given are

$$M = 367 \text{ W/m}^2$$

and

$$M_{bb} = 459 \text{ W/m}^2$$

Substituting these values into the equation,

$$\epsilon = M/M_{bb}$$

we have

$$\epsilon = (367 \text{ W/m}^2)/(456 \text{ W/m}^2)$$

or

$$\epsilon = 0.80$$

By definition, $\alpha = 1$ and $\epsilon = 1$ for a blackbody. In fact, emissivity is equal to absorptance for any graybody. For graybodies both will be less than one. We state this equality by the equation

$$\epsilon = \alpha \tag{8}$$

Equation (8) implies that good absorbers will also be good radiators and vice versa. A black object, in general, makes a poor heat shield. But a white or silvery object makes a good heat shield, since it neither absorbs nor emits energy readily.

Question 11. Two plates of aluminum, one shiny and the other painted black, are placed outside on snow-covered ground on a sunny winter day. What happens to the snow beneath the plates? (You could try this with two ice cubes by sprinkling a little carbon black on one and leaving them both in sunlight for a while.)

Problem 8. A blackbody at 1000 K has a radiant exitance of 5.7×10^4 W/m^2. What would be the radiant exitance of an object with absorptance $\alpha = 0.10$ at 1000 K?

The filament of an incandescent lamp may be considered approximately as a graybody. Actually tungsten does not absorb (or emit) all wavelengths equally well, but rather has a higher absorptance for radiation in the visible part of the spectrum than for the infrared. Over the visible spectrum, tungsten has an average absorptance or emissivity of about 0.3.

THE STEFAN-BOLTZMANN LAW

Experiment B-3 established the relation of radiated power to temperature. Further experiments, beyond the scope of this module, could also establish that the radiated power is proportional to the surface area of the radiating solid. Since the radiant exitance

is defined as power radiated ÷ area, it is independent of the area and is proportional to the fourth power of absolute temperature. The expression for radiant exitance is called the *Stefan-Boltzmann Law:*

$$M = \epsilon\sigma T_K^4 \quad (9)$$

where T_K is the absolute temperature and where $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot K^4$ is a constant called the *Stefan-Baltzmann Constant*. The dependence of radiated power on the fourth power of temperature means that the radiation will increase very rapidly with increasing temperature. Since the Stefan-Boltzmann Law gives the power per unit area, the total power radiated will be proportional to the surface area of the source.

Question 12. Suppose you have two equal amounts of iron; one is the shape of a sphere, the other stretched into a long, thin wire. If they are at the same temperature, how does the radiant exitance of one compare with that of the other?

Example Problem. The tungsten filament of a light bulb is operating at 2000 K. If the emissivity is taken as 0.30, what is the radiant exitance from the filament?

Solution. Given are T_K = 2000 K and ϵ = 0.30. Substituting values into the Stefan-Boltzmann Law,

$$\begin{aligned} M &= \epsilon\sigma T_K^4 \\ &= (0.3)(5.67 \times 10^{-8} \text{ W/m}^2 \\ &\quad \cdot \text{K}^4)(2000 \text{ K})^4 \end{aligned}$$

Taking the fourth power of the temperature, we write

$$\begin{aligned} M = {} & (0.3)(5.67 \times 10^{-8} \text{W/m}^2 \text{K}^4) \times \\ & (16 \times 10^{12} \text{K}^4) \end{aligned}$$

Simplifying, we have

$$M = 2.7 \times 10^5 \text{ W/m}^2$$

Problem 9. The tungsten filament of a light bulb is operating at 3000 K. If the emissivity of tungsten at this temperature is taken as 0.30, what is the radiant exitance from the filament?

SUMMARY

In this section you have learned that the pressure of a gas confined to a constant volume is proportional to the *absolute temperature* of the gas. Absolute temperature is expressed in kelvins and is related to the Celsius temperature by

$$T_K = T_C + 273$$

Because there is an increase of gas pressure with temperature, gas-filled bulbs usually have an internal pressure at room temperature which is less than one atmosphere.

You also learned that electrical resistance changes with temperature approximately according to the equation

$$R = R_o(1 + \alpha_T T_C)$$

where R is the resistance at celsius temperature T_C, R_o is the resistance at 0°C, and α_T is a constant for the material, called the *temperature coefficient of resistance*. This relation and the large value of α_T imply that the resistance of the filament of an incandescent lamp increases by a large factor when the lamp is turned on. This increase in resistance can actually be used to measure the operating temperature of the filament.

The tungsten filament approximates a *graybody*, which is an object that absorbs and emits the same fraction of incident power at all wavelengths. A *blackbody* absorbs and emits all of the incident power.

You also learned that *exitance* is the radiated power per unit area. *Absorptance* is defined as the fraction of the incident irradiance that is absorbed. The *emissivity* of any object is the ratio of the radiant exitance of the object to the radiant exitance of a blackbody at the same temperature. For blackbodies and graybodies the emissivity and the absorptance are equal.

Finally, you learned that the radiant exitance of a graybody, M, is related to the

absolute temperature by the *Stefan-Boltzmann Law*

$$M = \epsilon\sigma T_K^4$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$

OTHER APPLICATIONS OF THE PRINCIPLES

As you have already seen in one of the problems, the air in an automobile tire obeys the pressure-temperature relationship. For this reason, automobile manufacturers usually include two sets of tire pressure recommendations: one for cold tires and higher values for tires after they have run a while and become heated by friction.

The proportionality of pressure to absolute temperature can also be used to make an accurate type of thermometer called a *constant volume gas thermometer.* In this instrument a small volume of gas is enclosed in a bulb which is put into or on the substance whose temperature is to be measured. Then the pressure of the gas, which is easily measured, can be converted to a temperature reading. If helium is used as the filling gas, then low temperatures can be measured in this way because helium acts like an ideal gas even when very cold.

The Stefan-Boltzmann Law applies approximately to most solids. At least we generally can say that the power radiated by any object increases quite rapidly with its temperature. In the home typical devices which utilize the radiated power of a hot solid are electric toasters and electric broilers. In these devices, solid heating elements are heated to *incandescence* (red hot) by electric currents and energy is transferred to the food, primarily by radiation.

A scientific application of the Stefan-Boltzmann Law is to calculate the temperature of the sun's emitting surface. From the sample of the sun's power received here on earth, we can calculate the total power emitted, and then, knowing the sun's size, we can calculate its radiant exitance. The radiant exitance and the Stefan-Boltzmann Law allow us to calculate the temperature, treating the sun as a blackbody. The value obtained in this way is quite close to the value obtained from the sun's color, indicating that the sun is nearly a blackbody.

GOALS FOR SECTION C

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal. Answers appear immediately following these goals.

1. *Goal:* Know the definition of spectral radiant exitance.

   Item: Given that the radiant exitance of a tungsten filament is 5000 W/m^2 in the wavelength interval 3400-3600 nm, what is the approximate spectral radiant exitance at 3500 nm?

2. *Goal:* Know the relation of the total radiant exitance of a graybody to the graph of spectral radiant exitance versus wavelength.

   Item: Suppose you are given two curves of M_λ versus λ for a graybody, one for each of two temperatures, plotted on the same scale. The area under one curve is 4200 units and the area under the other curve is 700 units. Find the ratio of radiant exitances at the two temperatures.

3. *Goal:* Understand Wien's Displacement Law, and its relation to the Stefan-Boltzmann Law.

   Item: Sketch a graph of spectral radiant exitance versus wavelength for a graybody at a certain temperature. Suppose the absolute temperature of the object is doubled. Sketch a graph of spectral radiant exitance at this higher temperature, making the peak of the graph and the area under it roughly correct compared with the first graph.

4. *Goal:* Understand the concept and graph of relative luminosity.

   Item: Two light sources *appear* to be equally bright, but one is emitting at only 490 nm, while the other is emitting at only 515 nm. Which source is emitting more radiant power? How much more?

5. *Goal:* Know the definitions of luminous flux and lumens.

   Item: A helium-cadmium laser emits 15 milliwatts at the single wavelength of 442 nm. What luminous flux does the laser emit in lumens?

6. *Goal:* Understand the concept of luminous efficacy and the graph of this quantity versus temperature for a graybody.

   Item: One tungsten filament is operating at 2000 K while another with the same radiant flux is at 3000 K. What is the ratio of their luminous fluxes?

7. *Goal:* Know the definition of illuminance.

   Item: A 200-W incandescent lamp with reflector throws 2000 lm on the surface of a 4 ft by 8 ft pool table. What is the illuminance on the table top?

8. *Goal:* Know the illuminance as a function of distance from a point source emitting uniformly in all directions.

   Item: What is the illuminance at a distance of 2 m from a 60-W, 840-lm incandescent lamp?

Answers for Items Accompanying the Preceding Goals

1. 25 $W/m^2 \cdot nm$.
2. 6.
3. Graph (see Figure 19).
4. The source at 490 nm emits three times as much power.
5. 0.255 lm.
6. The 3000 K filament emits about three times the luminous flux of the 2000 K filament.
7. 62.5 footcandle.
8. 16.7 lux.

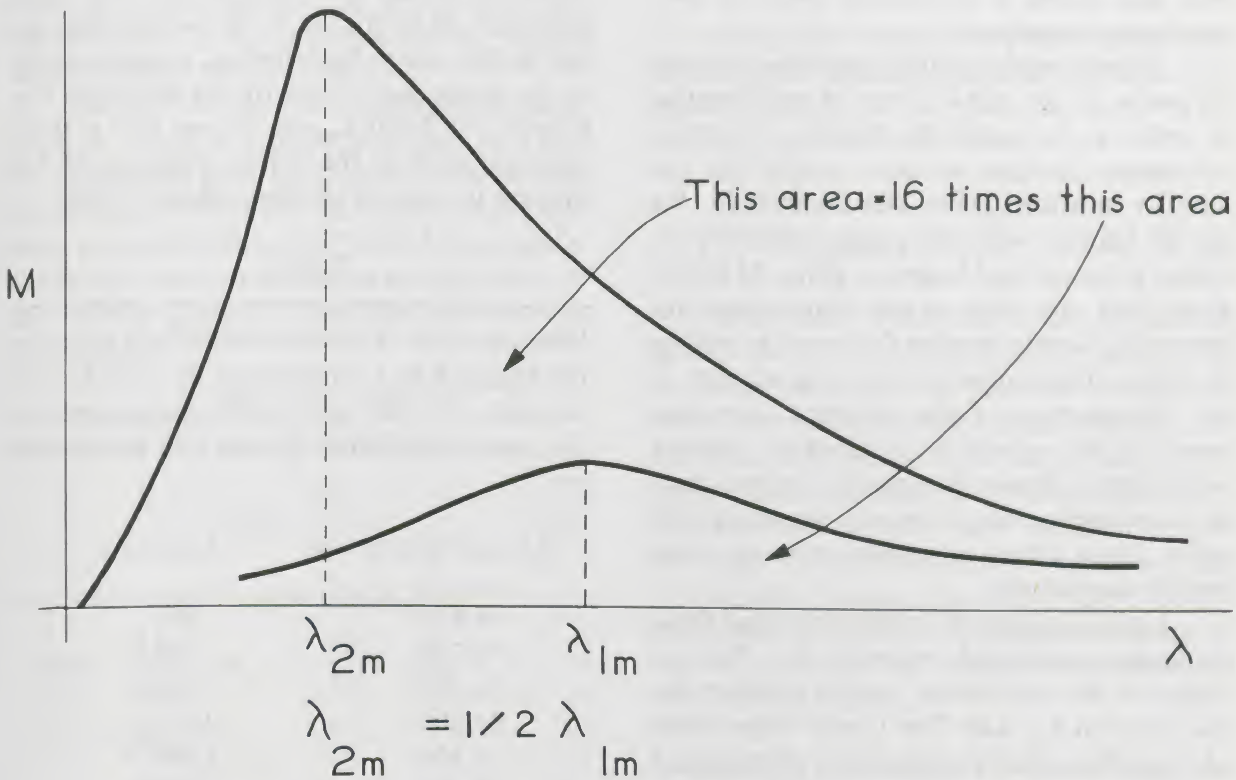


Figure 19. Graph for item 3.

SECTION C

An Analytical Approach

SPECTRAL RADIANT EXITANCE

We noted earlier that the spectrum of a light source determines both its color and its brightness, because of the eye's different sensitivities to different wavelengths. Therefore we are interested not only in the total power radiated by a graybody, but also in how that power is distributed over the electromagnetic spectrum.

A lamp must radiate a significant part of its power in the visible range of wavelengths, in order to be useful for lighting. Common experience can give us some insight into the relation between power and wavelength. We are all familiar with the process whereby an object is heated and begins to glow. At first it glows dull red, but as the temperature increases, the color changes from red to yellow to white. These observations suggest that as the temperature of the filament increases, more of the power is radiated at shorter wavelengths. Power is actually emitted over all wavelengths, but it is not equally distributed. More power is emitted at some wavelengths than others.

Let us consider the radiant exitance for a particular *wavelength interval,* $\Delta\lambda$. We can designate the *incremental radiant exitance* for that interval by ΔM. The Greek letter *delta* (Δ) is often used to indicate a *change* in a quantity or a short distance along a scale. Here $\Delta\lambda = \lambda_2 - \lambda_1$, which is a short distance along the wavelength scale, and ΔM is the power radiated per unit area of radiation surface for only that part of the spectrum between these two narrowly-separated wavelengths. In other words, M is all of the radiation whereas ΔM is just one small part of it. For example, the column headed ΔM in Table I shows measurements of the power radiated per unit area (of radiation surface) for tungsten at a temperature of 2450 K. These values of power per unit area are for wavelength intervals 200 nm wide. The column headed "λ" in Table I shows the average wavelength of each interval for which these incremental radiant emittance measurements were made.

The total radiant exitance, M, can be determined as the sum of *all* values of ΔM. From Table I you can see that we don't have all values of ΔM since in that case ΔM must reach zero at both ends of the list and they do not in this table. Nevertheless, measurements in the table cover most of the spectrum for tungsten at 2450 K, and we can find a good approximation to the radiant exitance, M, by forming the sum of all these values of ΔM.

Table I.

Measurements of incremental radiant exitance for tungsten at a temperature of 2450 K. For example, the ΔM of 16,400 corresponds to the wavelength interval from 400 nm to 600 nm.

ΔM (in W/m^2)	λ (in nm)
16,400	500
49,200	700
76,500	900
84,800	1100
79,300	1300
68,300	1500
57,400	1700
46,500	1900
35,500	2100
27,300	2300
19,100	2500
13,700	2700
10,900	2900
8,200	3100
5,500	3300
5,000	3500
4,500	3700
4,000	3900

Problem 10. Using the values of the incremental radiant exitance for 200 nm wavelength intervals in Table I, find an approximate value for the total radiant exitance of tungsten at 2450 K. How does this compare with the value given by the Stefan-Boltzmann Law with $\epsilon = 0.30$?

In equation form, what you just did in finding the sum of values of ΔM would be

$$M = (\Delta M)_1 + (\Delta M)_2 + (\Delta M)_3 + \ldots$$

where the dots mean to continue until all values have been included in the sum and the subscripts 1, 2, 3, . . . , refer to the various values of ΔM found in Table I.

We can change the appearance of each term in the sum without changing its value by multiplying each term by the number one. In this case, we choose the number one to be some quantity divided by itself. Let us pick a wavelength $\Delta\lambda$ corresponding to each value of ΔM as that quantity. Then the sum has the appearance

$$M = (\Delta M)_1 \frac{\Delta\lambda}{\Delta\lambda} + (\Delta M)_2 \frac{\Delta\lambda}{\Delta\lambda} + (\Delta M)_3 \frac{\Delta\lambda}{\Delta\lambda} + \ldots \qquad (10)$$

Let us now rewrite each term as follows:

$$M = \frac{\Delta M}{\Delta\lambda} \Delta\lambda_1 + \frac{\Delta M}{\Delta\lambda} \Delta\lambda_2 + \frac{\Delta M}{\Delta\lambda} \Delta\lambda_3 + \ldots \qquad (11)$$

The subscripts 1, 2, 3, . . . , indicate that each $\Delta\lambda$ corresponds to a particular ΔM. Although the values of ΔM in Table I are for equal values of $\Delta\lambda$ (200 nm), it is not absolutely necessary that $\Delta\lambda_1 = \Delta\lambda_2$, etc.

Equation (11) is written in order to define a useful quantity, the *spectral radiant exitance*, designated by M_λ,

$$[M_\lambda]_i = (\Delta M/\Delta\lambda)_i \qquad (12)$$

The subscript i (i = 1, 2, 3, . . .) means that the values of M_λ are found by dividing the ith value of ΔM by the corresponding value of $\Delta\lambda_i$.

Problem 11. Using values of incremental radiant exitance shown in Table I for wavelength intervals 200 nm wide for tungsten at 2450 K, make a table of values of spectral radiant exitance and corresponding values of wavelength. From this table construct a graph of spectral radiant exitance on the vertical axis and wavelength on the horizontal axis for tungsten at a temperature of 2450 K.

Using the definition of spectral radiant exitance, the sum expressed in Equation (11) can be written,

$$M = [M_\lambda]_1 \Delta\lambda_1 + [M_\lambda]_2 \Delta\lambda_2 + [M_\lambda]_3 \Delta\lambda_3 + \ldots \qquad (13)$$

Each term in this sum is the product of an M_λ and a corresponding $\Delta\lambda$. On a graph of M_λ versus λ, each such product corresponds to the area of a narrow rectangle, M_λ high and $\Delta\lambda$ wide. The sum of these little areas would approximate the total area "under the curve" (the area between the curve and the horizontal axis). The approximation improves as $\Delta\lambda$ gets smaller. Because the sum of these rectangular areas is also the total radiant exitance, M can be found by determining the area under a curve of M_λ versus λ.

Problem 12. In Problem 11, you constructed a graph of the spectral radiant exitance versus wavelength for tungsten at a temperature of 2450 K. Using that graph, construct a set of rectangles which cover the area under the curve, calculate the product $M_\lambda\Delta\lambda$ for each rectangle, and add them all up. How does this sum compare with the total radiant exitance you found in Problem 10?

It is possible to measure the power radiated per unit area (of a radiating surface) at a certain temperature for radiation in a

given wavelength interval. By dividing each such measurement by the corresponding value of the wavelength interval, $\Delta\lambda$, values of spectral radiant exitance, M_λ, can be determined. Each value of M_λ can be correlated with the wavelength λ at the center of its corresponding $\Delta\lambda$. Thus we can construct a graph of spectral radiant exitance versus wavelength for a radiating surface at a certain temperature. Figure 20 shows such a graph similar to the one you obtained in Problem 11.

You have seen that the area under the graph of M_λ versus λ gives the radiant exitance M. But in Section B you also learned that the radiant exitance is given by the Stefan-Boltzmann Law. Thus, for a graph of M_λ versus λ, we must have

$$\text{Area under a curve} = M = \epsilon\sigma T_K^4 \qquad (14)$$

WIEN'S DISPLACEMENT LAW

Figure 20 shows a typical spectral radiant exitance curve for a graybody at a single temperature. How would the curve change if the temperature of the body increased? You already know enough facts to give a reasonable answer. For one thing, Equation (14) tells you that the radiant exitance and therefore the area under the curve must increase. That is, the height of the curve must increase at higher temperatures. But as the temperature increases, the color shifts from red to yellow to white. This indicates that more power is emitted at shorter wavelengths for higher temperatures. Thus, the whole curve is displaced toward the left, toward shorter wavelengths. Figure 21, a plot of the spectral radiant exitance versus wavelength for a blackbody at several different temperatures, shows the effects of temperature.

A useful experimentally-determined equation gives the wavelength in nanometers which corresponds to the peak of the spectral radiant exitance curve for a given absolute temperature T_K. It is known as *Wien's Displacement Law*:

$$\lambda_m\ (\text{nm}) = (2.9 \times 10^6)T_K \qquad (15)$$

where λ_m is the wavelength at the peak.

This relationship indicates that as the temperature increases, the wavelength of maximum emission decreases. Typical values of λ_m are indicated in Figure 21.

Question 13. Astronomers classify stars according to color, from blue-white, to blue, white, yellow, orange, and red. What do these

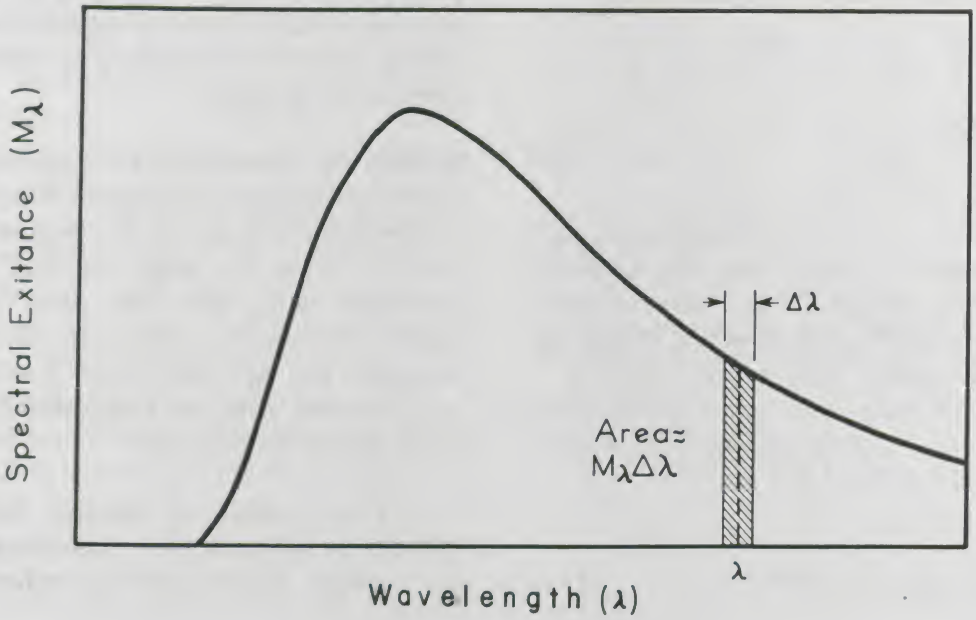


Figure 20.

colors mean in terms of the surface temperature of the stars to which the colors refer?

Example Problem. What is the wavelength at which maximum power per unit wavelength is emitted by the sun? (The surface temperature of the sun is around 6000 K.) In what region of the electromagnetic spectrum is this wavelength located?

Solution. Given is

$$T_K = 6000 \text{ K}$$

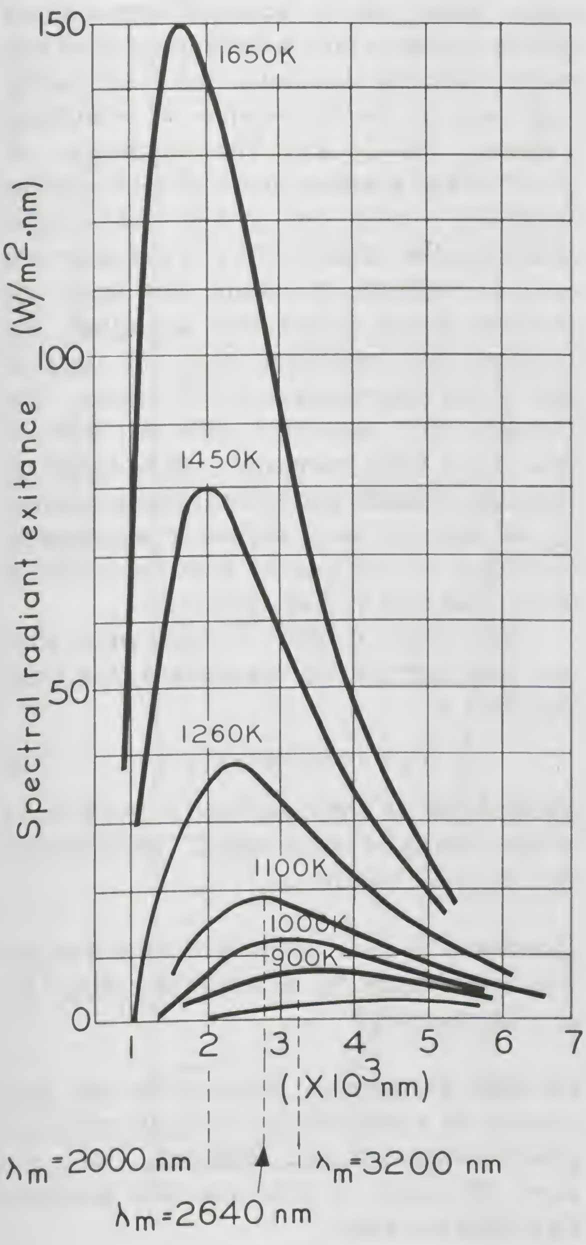


Figure 21.

Substituting given quantities into Wien's Displacement Law, we find that

$$\lambda_m = (2.9 \times 10^6 \text{ nm}\cdot\text{K})/(6 \times 10^3 \text{ K})$$

$$= 483 \text{ nm}$$

This wavelength is in the visible region of the spectrum and corresponds to a color of blue. The sun does not appear blue, however, since its energy is radiated almost evenly over the whole visible spectrum. Consequently, the sun appears almost white. (Also, the atmosphere tends to make the sun look yellow to us.)

Problem 13. What is the wavelength at which maximum power per unit wavelength is emitted by an incandescent lamp whose filament operates at 2500 K? In what region of the electromagnetic spectrum does this occur?

Question 14. What would the temperature of a substance need to be if the wavelength for the maximum emitted power per unit wavelength was in the microwave region of the spectrum? Use 1 mm for a typical microwave wavelength.

Problem 14. Using Wien's Displacement Law, calculate the value of wavelength for which the spectral radiant exitance would be maximum for tungsten at 2450 K. How does this result compare with the wavelength for which your graph of Problem 11 had its maximum value?

Problem 15. Assuming normal body temperature to be 37°C, at what wavelength does the human body have its maximum spectral radiant exitance?

LUMINOUS FLUX

The power (energy per unit time) carried away from the filament by electromagnetic waves is sometimes referred to as *radiant flux.* Flux simply means a flow or continuous movement. However, the radiant flux from an incandescent lamp is not a measure of the effectiveness of the device as a light source. In order to talk about light sources, the way the

human eye registers different light intensities, called *response*, must be taken into account. Radiant flux, evaluated with respect to its ability to produce the sensation of brightness in the human eye, is called *luminous flux*, or, by illumination engineers, simply *light*.

Only those wavelengths between 400 nm and 700 nm are visible to the eye. But the eye does not respond equally to all of those visible wavelengths. This is also the case with most other detectors of light. For the eye, the response curve starts at zero around 400 nm, rises to a maximum, and falls off to zero near 700 nm. Of course no two people are exactly alike, so that the response curve is different for different people. By testing the responses of a large number of people, averaging the results, and then plotting a graph so that the maximum response is unity, one obtains the *relative luminosity* curve for a standard observer, shown in Figure 22.

The quantity which is plotted on the vertical axis of Figure 22 is the relative sensitivity of the human eye to the corresponding wavelength, λ. This quantity is called the *spectral luminous efficiency*, K_λ.

The curve in Figure 22 is also called the *visibility curve*. One can see from the graph that the maximum K_λ occurs at about 555 nm in the yellow-green region of the spectrum. This maximum means that a radiant flux with a wavelength of 555 nm is more effective in producing a brightness sensation than an equal radiant flux at any other wavelength. In fact, one can see from the graph of Figure 22 that a given amount of radiant flux at a wavelength of 555 nm *appears* about three times as bright as the same amount of power radiated at a wavelength of 500 nm.

In determining the light output of an incandescent lamp, we must include the characteristic response of the human eye. The wattage rating of the lamp is an indication only of the input power and not of the useful light produced. The useful light output of a lamp depends not only on how much of the electric energy input is converted to radiation, but also upon the fraction of that radiation which is visible to the eye. *Luminous flux* is a measure of the extent to which radiation produced by a light source contains wavelengths which produce the sensation of brightness. The unit of luminous flux is called the *lumen* (lm).

The lumen is defined as the luminous flux in a certain cone of radiation from a standard source. The original standard source was a whale oil candle burning at a rate of 120 grains per hour, and the source brightness was called one *candela*. As you might suspect, the lumen defined in terms of this source depended on the particular whale oil candle being used. Recently, scientists defined a new source which can be accurately reproduced and the lumen is now defined in terms of this source. This new standard source is a radiating blackbody at the temperature of solidifying platinum (2045 K). The brightness of 1/600,000 of a square meter of this radiating blackbody is called one *candela*. The lumen is defined as the luminous flux in the same size cone of radiation as before, but from this different source. Using this definition, the luminous flux associated with *one watt* of light at 555 nanometers is 680 lumens. The luminous flux associated with one watt of light of any other wavelength can be found by using the visibility curve. The relative sensitivity of the eye to a particular wavelength, multiplied by 680 lumens, gives the luminous flux of one watt at that wavelength.

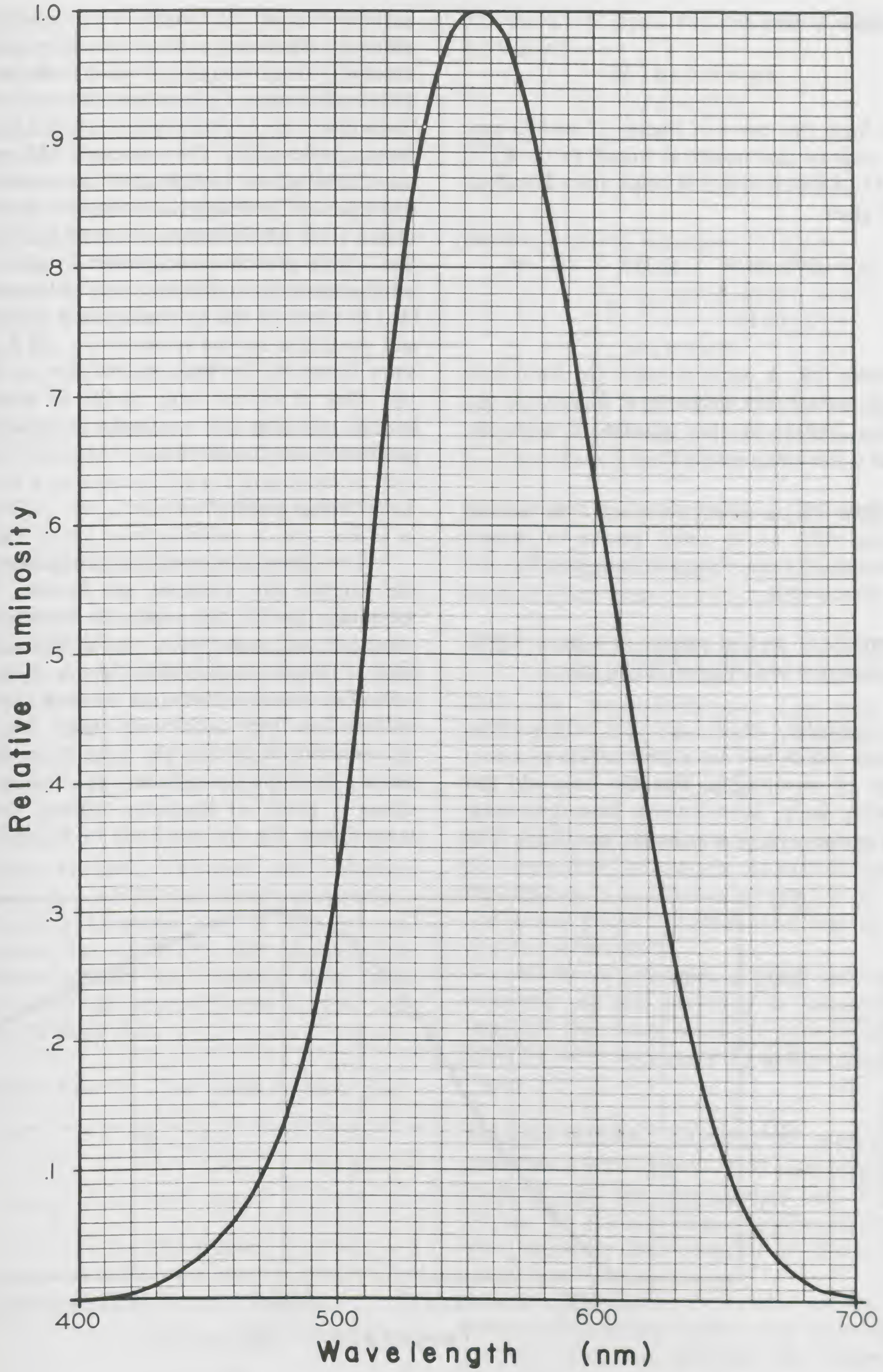
For values of radiated power other than one watt and for any wavelength, the luminous flux is

$$F_\lambda = (680\ \mathrm{lm/W})K_\lambda P_R \qquad (16)$$

where P_R is the power radiated in watts and λ is the particular wavelength. The luminous flux, F_λ, is in lumens.

Question 15. How much luminous flux per watt is produced by an ultraviolet lamp? By an x-ray machine?

Example Problem. A helium-neon gas laser operates at a wavelength of 633 nm and has a power output of one milliwatt. Using the curve of Figure 22, determine the luminous flux from this laser.


1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
Relative Luminosity
400
500
600
700
Wavelength (nm)
Figure 22.

Solution. Given is

$$P_R = 1 \times 10^{-3}\ \mathrm{W}$$

and from the curve of Figure 22 at 633 nm, the relative luminosity is found to be K_λ = 0.235. Substituting this value into Equation (16) gives

$$F_\lambda = 680\ \mathrm{lm/W} \times 0.235 \times 10^{-3}\ \mathrm{W}$$

$$= 0.160\ \mathrm{lm}$$

Problem 16. A sodium-vapor arc lamp produces radiant flux primarily at 589 nm. If the radiant power at this wavelength is 50 W, what is the luminous flux?

Problem 17. A 300-W mercury lamp is converting 40% of its input power to radiant power at 577 nm. Find the luminous flux for this wavelength.

LUMINOUS FLUX FROM SOURCE WITH EXTENDED WAVELENGTH RANGE

Actually, there are not many light sources which put out power within a narrow range of wavelengths, like the laser and the mercury lamp. Most sources have their energies spread over a wavelength spectrum. The incandescent lamp filament, as you have seen, produces significant radiant flux over a wide spectrum of wavelengths. We would like to be able to determine the luminous flux produced by radiation in a wavelength interval $\Delta\lambda$ near some wavelength λ. Then we could add up all the contributions of luminous flux over the spectrum of wavelengths produced by the source. This would give us the total luminous flux. This procedure is possible because we could express the radiated power of Equation (16) in terms of the spectral radiant exitance and use graphs to find values of M_λ and K_λ to sum. However, the process becomes so tedious that it is not very useful to pursue. Instead, we can use a simpler process for graybodies and blackbodies.

LUMINOUS EFFICACY

The characteristics of blackbodies and of the human eye response are known. The summing process can easily be done by a computer and put into a useful form. One such computation provides a graph of luminous flux versus temperature for each watt of radiant flux from a radiating object. We call the measure of lumens per watt of radiated power the *luminous efficacy*, e_L. Figure 23 shows a graph of luminous efficacy versus temperature for any graybody or blackbody.

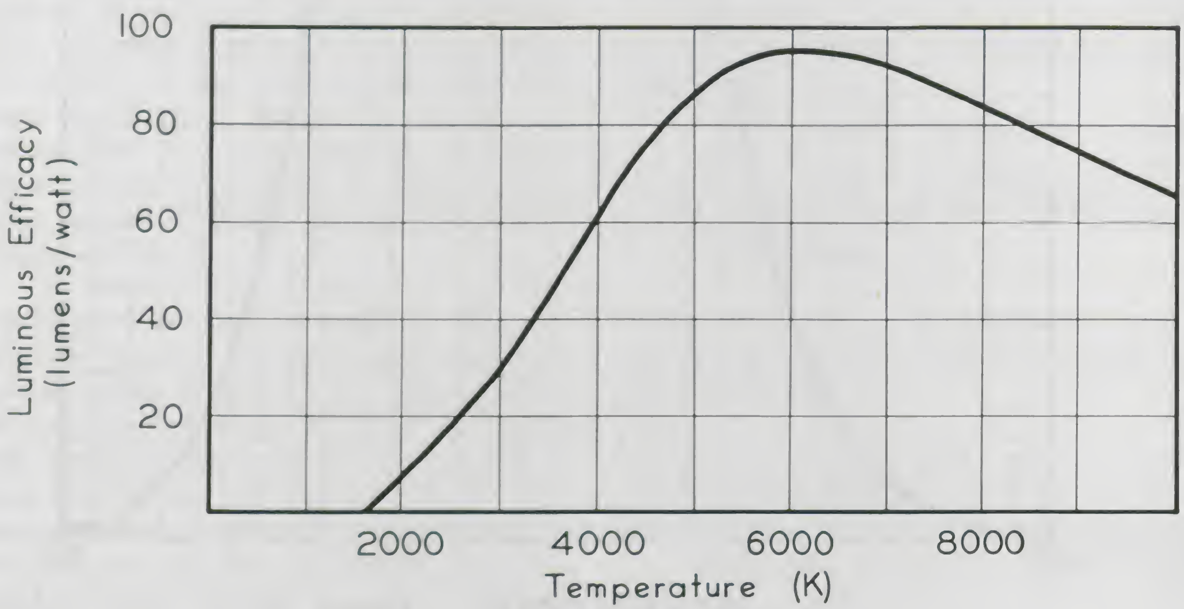


Figure 23.

Luminous efficacy is defined as the luminous flux from a source divided by the radiant flux from the source. This ratio is the same for a blackbody and a graybody because the distribution of radiated power over wavelength is the same for both. (Saying this another way, a blackbody is the same as a graybody whose emissivity is one, so anything which is true for a graybody with $\epsilon = 1$ is also true for a blackbody.) If e_L stands for luminous efficacy, then the definition may be stated by the equation

$$e_L = F/P_R \tag{17}$$

Since the *radiant flux* is a measure of the power radiated by an object and the *luminous flux* is a measure of the eye's response to that radiation, the *luminous efficacy* gives information about how effective a light source is for making things visible to humans.

Question 16. The spectrum of our sun peaks in the visible spectrum, whereas a tungsten filament at 2450 K peaks well into the infrared part of the spectrum. For an incandescent lamp, how does the amount of luminous flux at the red end of the visible spectrum compare with that from the blue end?

Example Problem. Determine the luminous efficacy of a 25-W incandescent lamp which produces a luminous flux of 266 lm and a radiant flux of 23 W. Also calculate the luminous efficacy of a 100-W lamp with 1700 lm of light output (luminous flux) and 82 W of radiant flux.

Solution. For the 25-W lamp we have given

$$P_R = 23 \text{ W}$$

and

$$F = 266 \text{ lumens}$$

Substituting into Equation (17) gives

$$e_L = 266 \text{ lm}/23 \text{ W}$$
$$= 11.6 \text{ lm/W}$$

for the 25-W bulb. For the 100-W bulb we have given

$$P_R = 82 \text{ W}$$

and

$$F = 1700 \text{ lm}$$

Substituting into Equation (17) gives

$$e_L = 1700 \text{ lm}/82 \text{ W}$$
$$= 21 \text{ lm/W}$$

for the 100-W bulb. Thus in this particular case, the 100-W bulb is a more efficient light producer than is the 25-W bulb.

LUMINOUS FLUX FROM A GRAYBODY

Equation (17) leads to another for the luminous flux from a graybody

$$F = e_L P_R = e_L a_s M_\lambda$$

Since the Stefan-Boltzmann Law may be stated as $M = \epsilon\sigma T_K^4$, we have for the flux F

$$F = e_L a_s \epsilon\sigma T_K^4 \tag{18}$$

This is the total luminous flux as a function of temperature, where a_s is the surface area of the source, ϵ is the emissivity, σ is the Stefan-Boltzmann constant, and T_K is the absolute temperature.

If the temperature, surface area, and emissivity of any graybody is known, the luminous flux from the body may be calculated by using Equation (18) and the graph of Figure 23.

Example Problem. Calculate the total luminous flux from a typical 60-W light bulb. Use 2500 K for the temperature and 7.7×10^{-5} m^2 for the area of the tungsten filament. The emissivity for tungsten is about 0.3.

Solution. As usual, it is best to list the known information before you start on a problem. For this problem, the following quantities are given:

$$T_K = 2500 \text{ K}$$
$$\epsilon = 0.3$$
$$a_s = 7.7 \times 10^{-5} \text{ m}^2$$

Other quantities needed for Equation (18) are σ and e_L. The constant σ is the Stefan-Boltzmann constant which has the value

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot K^4$$

The luminous efficacy, e_L, is found from the graph of Figure 23. At 2500 K the approximate value for e_L is read from the graph as

$$e_L = 18 \text{ lm/W}$$

We now substitute all of these values into Equation (18) for the total luminous flux and obtain

$$F = (18 \text{ lm/W})(7.7 \times 10^{-5} \text{m}^2)$$
$$\times (0.3)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)$$
$$\times (2500 \text{ K})^4$$

Performing the indicated arithmetic and simplifying the units gives

$$F = 920 \text{ lm}$$

This number matches fairly well the typical luminous flux for a 60-W incandescent lamp, as stated by the manufacturer.

Problem 18. Find the total luminous flux from a 25-W light bulb if the tungsten filament operates at 2300 K and has a surface area of 7.7×10^{-5} m^2. For this problem you may use an emissivity of 0.3 for tungsten.

FILAMENT TEMPERATURES

From the graph of Figure 23 one can see that the most efficient light-producing blackbody has a temperature of about 6000 K. At that temperature it would produce about 95 lm/W. Such high temperatures are impractical for an incandescent lamp filament because tungsten melts at 3665 K. Usually the operating temperature of a tungsten filament is between 2500 K and 3000 K. Longer-life light bulbs generally operate at lower temperatures to keep down filament evaporation. However, as the graph shows, less luminous flux per watt is produced at these lower temperatures. Thus, long life bulbs which operate at lower temperatures are less efficient and, for the same luminous flux, more costly to use.

OVERALL LUMINOUS EFFICACY

In speaking of light sources like the incandescent lamp, engineers often use the term *overall luminous efficacy* e_o, which is the total luminous flux divided by the electric power input:

$$e_o = F/P_{in} \qquad (19)$$

This is a measure of how efficient the input power is in producing useable light.

Example Problem. Calculate the overall luminous efficacy of a 25-W incandescent lamp which produces 266 lm of luminous flux.

Solution. Given are

$$F = 266 \text{ lumens}$$

and

$$P_{in} = 25 \text{ W}$$

Substitution into Equation (19) gives

$$e_o = 266 \text{ lm}/25 \text{ W} = 10.6 \text{ lm/W}$$

Problem 19. Calculate the *overall luminous efficacy* of a 100-W incandescent lamp which produces 1700 lm of luminous flux. What is the *luminous efficacy* of this lamp if the radiant flux is 92 W?

This ratio, e_o, will be smaller than e_L since the input power P_{in} is always somewhat larger than the radiated power P_R. This is because some of the input power is lost from the filament through the conduction of heat.

ILLUMINANCE

We all know that the amount of detail we see in a written page or in a scene depends on the amount of light available, or on the "illumination." To precisely describe the quantity of light striking a surface, we define the *illuminance*. Illuminance (E_L) is defined as the luminous flux F striking a surface divided by the area a_s of the surface.

$$E_L = F/a_s \tag{20}$$

where F is the total luminous flux and a_s is the area of the illuminated surface.

INVERSE SQUARE LAW

To be consistent with our units, a_s should be in square meters. In that case, E_L is in lumens per square meter or *lux* (1 lm/m^2 = 1 lx). However, it is also common to find the illuminated area expressed in square feet, in which case E_L is in lumens per square foot or *footcandles* (1 lm/ft^2 = 1 fc). If the luminous flux is emitted by a *small source* uniformly in all directions, the luminous flux at a distance r from the source is uniformly distributed over the surface of a sphere of radius r, centered on the source. As the luminous flux is measured farther from the source, it is spread out over a larger and larger sphere, a portion of which is shown in Figure 24. Since the surface area of a sphere of radius r is $4\pi r^2$, the illuminance at a distance r is

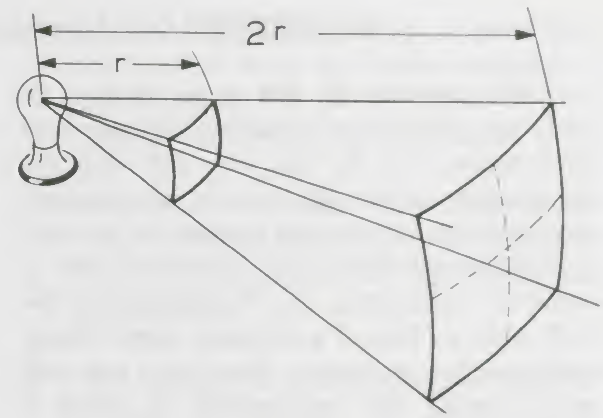


Figure 24.

$$E_L = F/4\pi r^2 \tag{21}$$

Remember this is the illuminance as a function of distance from a *small source*.
At twice the distance from a source, light is distributed over four times the area. The illuminance is, therefore, only one fourth as great at $2r$ as at r.

Question 17. A small source of light is moved away from a particular distance from a screen to twice as far away. By what factor does the illuminance change? What happens when the screen is moved three times as far?

EXPERIMENT C-1. Luminous Efficacy of A Tungsten Filament

The purpose of this experiment is to study the variation of luminous efficacy with temperature. To do this you will vary the temperature of the filament of an incandescent lamp by varying the current (as you did in Experiment B-3). The luminous flux is found by comparing the illuminance of the 25-W bulb to that of a standard lamp, whose luminous flux is known. The device you will use to make this comparison is called a *Bunsen Photometer.*

Connect the 25-W lamp to the power supply and voltmeter and ammeter as you did in Experiment B-3. Place the lamp on the end of the optical bench. Place the Bunsen photometer in the center of the optical bench and the standard lamp on the other end of the optical bench. Turn the standard lamp so that the side for which it is calibrated is toward the Bunsen photometer. The arrangement should now be as shown in Figure 25.

Plug the standard lamp into an AC outlet and set the current through the 25-W lamp to 0.14 A. Look into the front of the Bunsen photometer. You will see a split-screen view of the light coming from the two lamps. By changing the distance from the photometer to the lamps, the brightness on the two sides of the screen can be made equal. Adjust the distance until you think both sides of the screen are equally bright. When you have done this, the illuminance at the position of the photometer is the same from each lamp. In mathematical terms

$$E_{Lx} = E_{Ls}$$

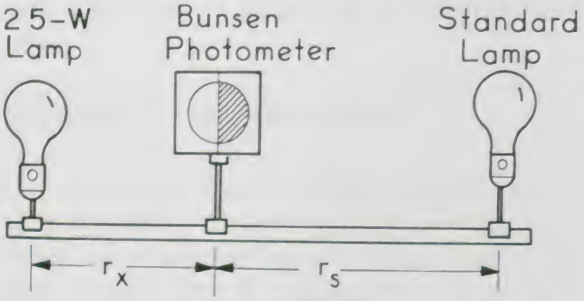


Figure 25.

where E_{Lx} is the illuminance from the 25-W lamp at the photometer and E_{Ls} is the illuminance from the standard lamp at the photometer. From Equation (21) E_{Lx} is given by

$$E_{Lx} = F_x / 4\pi r_x^2$$

where F_x is the luminous flux from the 25-W lamp and r_x is the distance from the 25-W lamp to the photometer.

The illuminance from the standard lamp at the photometer is

$$E_{Ls} = F_s / 4\pi r_s^2$$

where F_s is the known luminous flux from the standard lamp and r_s is the distance from the standard lamp to the photometer.

Substituting the above two equations into the first equation gives

$$\frac{F_x}{4\pi r_x^2} = \frac{F_s}{4\pi r_s^2}$$

Solving the above equation for F_x gives

$$F_x = F_s (r_x^2 / r_s^2)$$

Using the above equation, the luminous flux from the 25-W lamp can be calculated from the luminous flux for the standard lamp and the distances from the photometer to the lamps.

To use this to find the luminous efficacy proceed as follows:

1. Record the value of the luminous flux from the standard lamp.

2. With the current through the 25-W lamp set at 0.14 A, adjust the distance to the photometer for equal illuminance on each side. Record the current and voltage for the 25-W lamp and the distances r_x and r_s.

3. Repeat Step 2 in steps of 0.02 A until a maximum of 0.26 A is reached.

4. Calculate and record the electrical power input for each current setting, using the equation

$$P = IV$$

5. Using the equation for F_x, calculate and record the luminous flux for each setting.

6. Using the equation

$$e_o = (F_x)/(P_{in})$$

to calculate the luminous efficacy for each of the current settings. Assume that for a 25-W lamp the power radiated is the same as the power input. Record your values of e_o in the work sheet.

7. Refer to your data from Experiment B-3. Record the same absolute temperature for each current setting used in this experiment.

8. Plot a graph of overall luminous efficacy e_o on the vertical axis and absolute temperature T on the horizontal axis.

9. How does your graph compare with that shown in Figure 23?

NUMERICAL VALUES FOR ILLUMINANCE

Example Problem. Calculate the luminous flux which must be incident on a desk top of area $a_s = 0.6\ \mathrm{m}^2$ in order for the desk to have an illuminance of 500 lx.

Solution. Given are

$$E_L = 500\ \mathrm{lx} = 500\ \mathrm{lm/m^2}$$

and

$$a_s = 0.6\ \mathrm{m}^2$$

Equation (20) rearranged to solve for F becomes

$$F = E_L \times a_s$$

Substituting into this equation yields

$$F = 500\ \mathrm{lm/m^2} \times 0.6\ \mathrm{m}^2$$

$$= 300\ \mathrm{lm}$$

Problem 20. If an office is to be lighted at a level of 75 fc (footcandles), calculate the luminous flux needed to meet this standard for a desk with an area of 6 ft^2.

Example Problem. In order to meet the suggested minimun lighting of 75 fc on the center of a desk top, how high above that point should a 100-W lamp be placed? The luminous flux from a 100-W lamp is 1700 lm. (Assume that the lamp can be treated as a small source of light.)

Solution. Given are

$$F = 1700\ \mathrm{lm}$$

and

$$E_L = 75\ \mathrm{fc} = 75\ \mathrm{lm/ft^2}$$

Rearranging Equation (21) to solve for r^2 gives

$$r^2 = F/(4\pi E_L)$$

Taking the square root of both sides, we obtain

$$r = \sqrt{F/4\pi E_L}$$

Substituting into this equation gives

$$r = \sqrt{\frac{1700\ \mathrm{lm}}{(4)(3.14)(75\ \mathrm{lm/ft^2})}}$$

$$= 1.34\ \mathrm{ft}$$

Problem 21. The minimum illuminance for illumination in a machine shop should be 100 fc. How far above the working area should a 300-W lamp be placed in order to meet this standard? Assume that the luminous flux of a 300-W lamp is 6000 lm.

Example Problem. Lighted entrance ways to buildings should have a minimum of 5 fc of illuminance.* What is the minimum wattage (power rating) of the incandescent lamp that should be used if the lamp is located 10 ft above the entrance floor? For approximate luminous flux ratings on various sizes of lamps, use Table II.

Solution. Given are

$$E_L = 5\ \mathrm{fc} = 5\ \mathrm{lm/ft^2}$$

Table II.

Appropriate luminous flux ratings of incandescent lamps

Wattage (W)	Luminous Flux (lm)
25	260
60	840
75	1100
100	1700
200	4000
300	6000
500	10000

*Recommended values from the Illuminating Engineering Society (I.E.S.) Lighting Handbook, 4th edition.

Table III.

Some typical values for natural illuminance

MODE OF ILLUMINATION	lx (lm/m^2)	fc (lm/ft^2)
Sunlight plus skylight (maximum).	110,000	10,000
Sunlight plus skylight (dull day).	1,100	100
Interiors - daylight.	220	20
Full Moonlight.	0.22	0.020

and

$$r = 10 \text{ ft}$$

Rearranging Equation (21) to solve for F gives

$$F = 4\pi r^2 E_L$$

Substituting values into this equation yields

$$F = (4)(3.14)(10 \text{ ft})^2 (5 \text{ lm/ft}^2)$$

Simplifying gives

$$F = 2 \times 3.14 \times 10^3 \text{ lm} = 6300 \text{ lumens}$$

Thus the 300-W bulb will almost meet the minimum lighting standard.

Problem 22. Parking areas for automobiles should have a minimum illuminance of about 10 fc. What size lamp should be used in a fixture which is 12 ft above the lot?

Table III gives some approximate illuminance values due to natural lighting, and Table IV gives some recommended artificial lighting values for various visual tasks.

Question 18. Why would it be wrong to use Equation (21) to calculate the illuminance from a lamp which has a reflector? What about a long, tubular fluorescent lamp?

SUMMARY

In this final section of the module, you have learned that the distribution of power in the spectrum of a radiating source is given by the spectral radiant exitance, which is defined by

$$M_\lambda = \Delta M / \Delta\lambda$$

where ΔM is the small amount of radiant exitance in the small wavelength interval $\Delta\lambda$. In general, M_λ varies with λ, the center value of $\Delta\lambda$. For a graybody the graph of M_λ versus λ looks typically like those curves shown in Figure 21. The total radiant exitance of a graybody at any temperature is equal to the area under the graph of M_λ versus λ.

The wavelength λ_m at which the maximum value of M_λ occurs (the peak of the curve) is given by *Wein's Displacement Law*:

$$\lambda_m = (2.9 \times 10^6)/(T_K)$$

This equation predicts the shift of the spectrum of a graybody to shorter wavelengths as its absolute temperature increases.

You learned that the brightness of a light source depends not just on the power radiated (*radiant flux*) but also on how that power is distributed through the spectrum with respect to the response of the eye. The response of the eye is indicated by the *relative luminosity curve* on the graph of *spectral luminous efficiency* K_λ, shown in Figure 22. The ability of *radiant flux* P_R of a single wavelength to produce the brightness sensation in the eye is expressed by the *luminous flux* F_λ in lumens, given by the formula

$$F_\lambda = (680 \text{ lm/W}) K_\lambda P_R$$

Here the value of K_λ for the wavelength of the radiant flux P_R must be used.

Table IV.

Recommended* illuminance from artificial lighting.

AREA TO BE ILLUMINATED	ILLUMINANCE lx	fc
Auditorium during presentation	1.1	0.1
Exterior building and yard	55	5
Lobbies, store circulation areas, entrances	330	30
General home lighting	550	50
Classrooms	770	70
Offices, general	1,100	100
Fine bench work in machine shops	5,500	500
Dental chair	11,000	1,000
Fine inspection work in manufacturing plants	22,000	2,000

The *total luminous flux F* for a source which emits a band of wavelengths can be found by a process of summing the contributions of flux over many small wavelength intervals. This summation has been done for graybodies at various temperatures, and the results are defined by the *luminous efficacy curve* of Figure 23. Luminous efficacy e_L is defined as the luminous flux divided by the radiant flux from the source, or

*Recommended values from the Illuminating Engineering Society (I.E.S.) Lighting Handbook, 4th edition. These U.S. minimum standards are from five to ten times larger than those used by the rest of the world.

$$e_L = F/P_R$$

Finally, you learned that the basic working measure of lighting on an area a_s is the *illuminance* E_L, defined as the luminous flux per unit area, or

$$E_L = F/a_s$$

For a small source emitting uniformly in all directions, the illuminance follows an inverse square law:

$$E_L = F/4\pi r^2$$

EXPERIMENT A-1
Work Sheets

Name________________________

Part I

1. ________________________
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2. ________________________
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3. ________________________
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4. ________________________
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5. ________________________
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6. ________________________
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7. ________________________
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8. ________________________
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9. ________________________
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10. ________________________
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11. ________________________
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12. ________________________
____
____

13. ________________________
____
____

Part II

1.

Voltage (volts)	Current (amperes)	Color (name and number)
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		

2. ________________________

3. ________________________
____
____

4. ________________________
____
____

5. ______________________________

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____

Part III

1. ______________________________

____

2. ______________________________

____

3. ______________________________

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EXPERIMENT A-2
Work Sheets

Name______________________________

Part I

1. ______________________________

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2. ______________________________

3. ______________________________

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4. ______________________________

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5. ______________________________

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6. ______________________________

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7. ______________________________

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8. ______________________________

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Part II

1. ______________________________

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2. ______________________________

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COMPUTATION SHEET

EXPERIMENT B-1
Work Sheets

Name______________________________

	Temperature (°C)	Pressure (lb/in^2)
1.		
2.		
3.		

6. ______________________°C

7. $P =$ ______________________

4. Graph of Pressure vs Temperature

COMPUTATION SHEET

EXPERIMENT B-2
Work Sheets

Name________________________________

Temperature (°C)	Resistance (ohms)

1. ________________________________

2. ________________________________

3. Slope = ____________________

4. ________________________________

   ____

5. ________________________________

7. ________________________________

8. ________________________________

   ____

   ____

COMPUTATION SHEET

EXPERIMENT B-3
Work Sheets

Name______________________________

1. R_o = ______________________

2.

I (amps)	V (volts)	$P = IV$ (watts)	$R = V/I$ (ohms)	R/R_o	T (kelvins)

3. Plot a graph of P and T_k.

Method I

1.

Input power	T_k	T_k^2	T_k^3	T_k^4

2. Plot graphs of P and T_k^n.

3. ____________________________________

____

____

Method II

1. Plot a graph of log P and log T_k.

2. Slope = ________________________

3. ____________________________________

EXPERIMENT C-1
Work Sheets

Name______________________________

1. F_s = ______________________ lm.

I (a)	V (v)	r_x (m)	r_s (m)	P (w)	F_x (lm)	e (lm/w)	T (K)

9. ____________________________________ ____________________________________

____________________________________ ____________________________________

____________________________________ ____________________________________

____________________________________ ____________________________________

absorptance (α): The ratio of absorbed radiation to the total radiation striking a surface (the incident radiation). For most objects, the absorptance depends on the wavelength of the radiation.

$$\alpha = \frac{\text{absorbed radiation}}{\text{incident radiation}}$$

blackbody: An object which absorbs all radiation falling on it. That is, an object which has an absorptance $\alpha = 1$ for all wavelengths.

candela (cd): A unit of brightness, equal to the brightness of 1/60 of a square centimeter of a blackbody radiating at 2046 K (the temperature at which liquid platinum solidifies).

emissivity (ϵ): The ratio of the radiant exitance of a body to the radiant exitance of a blackbody at the same temperature. For a blackbody, $\epsilon = 1$, and $\epsilon < 1$ for all other radiators.

$$\epsilon = \frac{\text{radiant exitance of object at } T \text{ (K)}}{\text{radiant exitance of blackbody at } T \text{ (K)}}$$

flux: The flow of radiation energy.

footcandle (fc): A unit of illuminance equal to one lumen per square foot. Although not an SI unit, it is widely used. One footcandle equals 10.76 lux.

graybody: An object whose emissivity ϵ is less than one, but is constant for all wavelengths of radiation.

illuminance (E_L): The luminous flux per unit area striking a surface which is being illuminated.

incremental luminous flux (ΔF_λ): The fraction of the total luminous flux which corresponds to a wavelength interval centered on some specified wavelength.

incremental radiant exitance (ΔM): The fraction of the total radiant exitance which corresponds to a wavelength interval centered on some specified wavelength.

irradiance, absorbed (E_{abs}): The radiant flux absorbed by a surface divided by the area of the surface.

irradiance, incident (E_{inc}): The radiant flux striking a surface divided by the area of the surface.

$$E = \frac{\text{radiant flux}}{\text{surface area}}$$

lumen (lm): The fraction $1/4\pi$ of the total luminous flux emitted by a light source (of very small size) which has an intensity of one candela.

luminous efficacy (e_L): The ratio of luminous flux to the radiant flux. The luminous efficacy measures the effectiveness of a source in producing a visual sensation of light.

$$e_L = \frac{\text{luminous flux}}{\text{radiant flux}}$$

luminous flux (F): The rate of flow of light (radiation visible to the eye) energy per unit time.

lux (lx): The SI unit of illuminance, equal to one lumen per square meter of illuminated surface.

nanometer (nm): An SI unit of length used to describe wavelengths of light. One nanometer is equal to 10^{-9} meters.

phosphor: A solid material which can absorb and re-emit light. Usually the re-emitted light is of longer wavelength than the absorbed light.

radiant exitance (M): The radiant flux emitted by a surface divided by the area of the surface.

$$M = \frac{\text{radiant flux emitted}}{\text{surface area}}$$

radiant flux (P): The energy per unit time (power) radiated by a surface.

relative luminosity (K_λ): The apparent (to the human eye) brightness of a surface. An equivalent, and more often used, term is spectral luminous efficiency.

spectral luminous efficiency (K_λ): The relative sensitivity of the human eye to radiation of wavelength λ. The maximum value of K_λ ($K_\lambda = 1$) is for light of wavelength 555 nm.

spectral radiant exitance (M_λ): The power per unit area radiated by a source divided by the wavelength interval.

$$M_\lambda = \frac{\text{incremental radiant exitance } (\Delta M)}{\text{wavelength interval } (\Delta\lambda)}$$

temperature coefficient of resistance (α_T): The fractional change in resistance divided by the corresponding change in temperature. If T_1 and R_1 are initial values of temperature and resistance, and T_2, R_2 are final values, then

$$\alpha_T = \frac{R_2 - R_1}{R_1} \cdot \frac{1}{T_2 - T_1}$$

Usually T_1 is a reference temperature (often 0°C) and

$$\alpha_T = \frac{R - R_o}{R_o} \cdot \frac{1}{T}$$

where R is the resistance at the temperature T and R_o is the resistance at 0°C.

visibility curve: A plot of the eye's sensitivity to light (the spectral luminous efficiency K_λ) against wavelength λ.

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